DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO
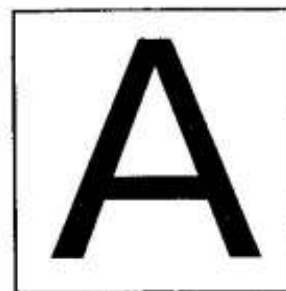
T.B.C. : P-TSB-N-NND

Test Booklet Series

A

TEST BOOKLET

MATHEMATICS

Paper – III

Time Allowed : Two Hours

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES **NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. **Please note that it is the candidate's responsibility to encode and fill in the Roll number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR answer sheet. Any omission/discrepancy will render the answer sheet liable for rejection.**

3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. **DO NOT** write **anything else** on the Test Booklet.

4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.

5. You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.

6. All items carry equal marks.

7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.

8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator **only the Answer Sheet.** You are permitted to take away with you the Test Booklet.

9. Sheets for rough work are appended in the Test Booklet at the end.

10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i)   There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0.33)** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. If $A = \{1, 2, 3\}$ and $B = \{1, 2\}$ and $C = \{4, 5, 6\}$, then what is the number of elements in the set $A \times B \times C$?

    (a) 8

    (b) 9

    (c) 15

    (d) 18

2. There are unlimited number of identical balls of four different colours. How many arrangements of at most 6 balls can be made by using them ? [to be arranged in a single row only]

    (a) 1365

    (b) 2730

    (c) 5460

    (d) None of the above

3. Let Z be the set of integers and 'o' be a binary operation of Z defined by $a \text{ o } b = a + b - ab$ for all $a, b \in Z$. What is the inverse of an element $a (\neq 1) \in Z$?

    (a) $a/(a - 1)$

    (b) $a/(1 - a)$

    (c) $(a - 1)/a$

    (d) None of the above

4. If $x^2 - x + 1 = 0$, then what is the value of

    $$\sum_{n=1}^{5} \left(x^n + \frac{1}{x^n} \right)^2 ?$$

    (a) 8

    (b) 10

    (c) 12

    (d) None of the above

5. If n is the positive integer such that
    $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots a_{2n} x^{2n}$,
    then what is the value of $\displaystyle\sum_{r=0}^{2n} a_r$?

    (a) 0

    (b) 3^{n+1}

    (c) 3^{n-1}

    (d) 3^n

6. If $(x + 1)(x + 2)(x + 3)(x + 6) = 3x^2$, then the equation has

    (a) all imaginary roots

    (b) all real roots

    (c) two rational and two irrational roots

    (d) two imaginary and two irrational roots

7. If $z = \dfrac{1+i}{1-i}$, where $i = \sqrt{-1}$, then what is the value of $\dfrac{z^3 + z}{z^2}$?

    (a) i

    (b) 0

    (c) 1

    (d) $-i$

8. If f(x), g(x) and h(x) are three polynomials of degree 2 and $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

    then $\Delta(x)$ is a polynomial of degree (dashes denote the differentiation)

    (a) 2

    (b) 3

    (c) 0

    (d) at most 3

9. If α, β are the roots of $x^2 + px - q = 0$ and γ, δ are the roots of $x^2 - px + q = 0$, then what is $(\beta + \gamma)(\beta + \delta)$ equal to ?

(a) 0

(b) $p + q$

(c) $2q$

(d) $p - q$

10. The graph of the quadratic polynomial $f(x) = (x - a)(x - b)$ where a, b > 0 and a ≠ b, then the graph does *not* pass through

(a) first quadrant

(b) second quadrant

(c) third quadrant

(d) fourth quadrant

11. There are two sequences whose n^{th} term is given by

1. $a_n = a_{n-1} + a_{n-2}$ where n > 2 with $a_1 = 1$, $a_2 = 2$.

2. $a_n = 2a_{n-1}$ with $a_1 = 2$, where n > 1.

Which one of the following is correct in respect of the above ?

(a) 1 and 2 are both APs

(b) 1 only is in AP

(c) 2 only is in AP

(d) Neither 1 nor 2 is in AP

12. The product of n positive different real numbers is 1. The sum of these n real numbers

(a) must be an integer

(b) less than n

(c) $n(n + 1)$

(d) never less than n

13. What is the sum of the infinite series

$$1 + \frac{2 \times 9}{10} + \frac{3 \times 9^2}{10^2} + \frac{4 \times 9^3}{10^3} + \ldots ?$$

(a) 10

(b) 100

(c) 1/100

(d) Does not exist

14. If A is a non-singular square matrix such that $A^2 - A + I = 0$, then what is A^{-1} equal to ? (I is identity matrix of same order as A)

(a) I

(b) $I + A$

(c) A^2

(d) $I - A$

15. A group of order 6 has an element of order

(a) 2

(b) 4

(c) 5

(d) 7

16. Consider the following statements in respect of the relation between two integers m and n as m is related to n if $(m - n)$ is divisible by 5.

   I. The relation is an equivalence relation.

   II. The collection of integers related to 1 and the collection of integers related to 2 are disjoint.

   Which of the above statements is/are correct ?

   (a) I only
   (b) II only
   (c) Both I and II
   (d) Neither I nor II

17. The value of p, for which the quadratic equation $x^2 - 4px + 4p(p - 1) = 0$ possesses roots of opposite sign, lies in

   (a) $(0, 1)$
   (b) $(-\infty, 0)$
   (c) $(1, 4)$
   (d) $(1, \infty)$

18. Let U be the set of 4-digit numbers that can be formed with the digits 2, 3, 6, 7, 8, 9, each digit being used only once in each number and V be the set of 4-digit numbers that can be formed with the digits 9, 8, 7, 1, 4, 5, each digit being used only once in each number.

   Consider the following in respect of the above :

   1. The number of elements of
      $U \cup V = n^4 - 280$ for some positive integer n.

   2. The number of elements of
      $U + 280 = m^2/10$ for some positive integer m.

   Which of the above statements is/are correct ?

   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

19. A five-digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

   (a) 216
   (b) 240
   (c) 600
   (d) 3125

20. The value(s) of λ (real or complex) for which the following system of linear equations $(\lambda - \cos 0) x - (\sin 0) y = 0$ and $(\sin 0) x + (\lambda - \cos 0) y = 0$ admits a solution such that at least one of x, y is different from zero is/are

   (a) $\cos 0 \pm i \sin 0$
   (b) $\cos 20 \pm i \sin 20$
   (c) 0
   (d) 1

   where $i = \sqrt{-1}$.

21. Let A and B be two 3×3 matrices whose determinants are 2 and 4 respectively. What is the value of $\det(adj(AB))$?

   (a) 6
   (b) 8
   (c) 64
   (d) 512

22. Let A be a 3×3 matrix such that $A^T = A^{-1}$. Let B be another 3×3 matrix whose determinant is 3. What is the determinant of the matrix $A^T B^3 A$?

   (a) 3
   (b) 6
   (c) 9
   (d) 27

23. Which one of the following is correct ?

   (a) $6 \cos 20° - 8 \cos^3 20° - 1 = 0$

   (b) $8 \sin^3 10° - 6 \sin 10° + 1 = 0$

   (c) $8 \cos 20° - 6 \cos^3 20° + 1 = 0$

   (d) $6 \sin^3 30° - 8 \sin 10° + 1 = 0$

24. If $x = \cos 15°$, then which one of the following is correct ?

   (a) $4x^2 - 2 + \sqrt{3} = 0$

   (b) $8x^3 - 6x + \sqrt{2} = 0$

   (c) $8x^3 + 6x - \sqrt{2} = 0$

   (d) $16x^4 - 16x^2 + 1 = 0$

25. If $\cos(\alpha + \beta) = 0$, then what is $\operatorname{cosec}(\alpha - \beta)$ equal to ?

   (a) $\cos 2\beta$

   (b) $\sec 2\beta$ only

   (c) $-\sec 2\beta$ only

   (d) $\pm \sec 2\beta$

26. A triangle of area 14 square units has two sides of lengths 7 units and 12 units. What is the angle between the two sides ?

   (a) $\sec^{-1}\left(\dfrac{3}{2\sqrt{2}}\right)$

   (b) $\sin^{-1}\left(\dfrac{2}{3}\right)$

   (c) $\tan^{-1}\left(2\sqrt{2}\right)$

   (d) $\operatorname{cosec}^{-1}(2)$

27. If $\cot^2 x + \operatorname{cosec} x - a = 0$ has at least one solution, then the complete set of values of a belongs to

   (a) $[-1, \infty)$

   (b) $[-3, -2]$

   (c) $(-2, -1)$

   (d)   None of the above

28. If $\tan \alpha = \dfrac{1 + \sqrt{1 + \sin 2\theta}}{1 - \sqrt{1 - \sin 2\theta}}$, where $0 < \theta < \dfrac{\pi}{4}$, then what is the value of α ?

   (a) $\dfrac{\pi}{2} - \dfrac{\theta}{2}$

   (b) $\dfrac{\pi}{4} + \dfrac{\theta}{2}$

   (c) $\pi + \theta$

   (d)   None of the above

29. Consider the following statements :

   1   For any real number θ, $\sin^6 \theta + \cos^6 \theta \leq 1$.

   2.   If x and y are non-zero real numbers such that $x^2 + y^2 = 1$, then $x^4 \leq 1/2$ and $y^4 \leq 1/2$

   Which of the above statements is/are correct ?

   (a)   1 only

   (b)   2 only

   (c)   Both 1 and 2

   (d)   Neither 1 nor 2

30. Let $0 < \theta < 90°$ be such that $\tan \theta = \sqrt{2}$. Consider the following statements :

   1.   There exist distinct integers x, y, z such that $\sec^2 \theta = x^2 + y^2 + z^2 - xy - yz - zx$.

   2.   It is impossible to find integers x, y, z $(x \neq y, y \neq z, z \neq x)$ such that $\tan^2 \theta = x^3 + y^3 + z^3 - 3xyz$.

   Which of the above statements is/are correct ?

   (a)   1 only

   (b)   2 only

   (c)   Both 1 and 2

   (d)   Neither 1 nor 2

31. Suppose the figure 'A' is obtained by joining the points (2, 2), (6, 5), (10, 2). The horizontal line segment has end points (3, 4) and (x, y). Which one of the following is correct ?

(a) x = 7 and y = 4

(b) x = 3 and y = 4

(c) x = 9 and y = 4

(d) x = 6 and y = 4

32. What is the angle made by the tangent to the parabola $y^2 = 4x$ at the point $\left(\dfrac{1}{4}, 1\right)$ with the y-axis ?

(a) $\tan^{-1} 2$

(b) $45°$

(c) $\cot^{-1} 2$

(d) $60°$

33. What is the area of the triangle formed by the lines $\dfrac{x}{3} - \dfrac{y}{2} = 1$, $\dfrac{x}{4} - \dfrac{y}{6} = 1$ and the x-axis ?

(a) 1/8 square units

(b) 1/4 square units

(c) 1/5 square units

(d) 3/5 square units

34. A region in the xy-plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line y = 0. If the point (a, a + 1) lies in the interior of the region, then

(a) $a \in (-5, -3)$

(b) $a \in (-\infty, -1) \cup (3, \infty)$

(c) $a \in (3, 5)$

(d) $a \in (-1, 3)$

35. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $2x - 3y = 0$, then what is the value of c ?

(a) 1

(b) -1

(c) 3

(d) -3

36. If the circles $x^2 + y^2 + 4x + k = 0$ and $x^2 + y^2 + 4y + k = 0$ touch each other, then what is the value of k ?

(a) 1

(b) 2

(c) 4

(d) 18

37. If the point (2, k) is at unit distance from the line $3x - 4y + 1 = 0$, then the values of k are

(a) 1/2 or 3

(b) 1 or 1/2

(c) 1, -1

(d) 0, 1

38. At the point $(0, 7/8)$, the line joining $(2, 1)$ and $(-4, a)$ is trisected. What is the value of 'a' ?

(a) $13/16$

(b) $5/16$

(c) $13/8$

(d) None of the above

39. What is the argument of $-\sqrt{3}-i$?

(a) $30°$

(b) $150°$

(c) $210°$

(d) $240°$

40. If two straight lines $x \cos \alpha + y \sin \alpha = 1$ and $x \sin \beta + y \cos \beta = 1$ are perpendicular, then which one of the following statements follows logically ?

(a) $\alpha = \beta$

(b) $\alpha - \beta = 90°$ only

(c) $|\alpha - \beta| = 90°$

(d) None of the above

41. Let X be a set of real numbers for which the function $f(x) = \sin(x - |x|)$, $x \in R$ is continuous. Then X is equal to

(a) $R - Z$

(b) Z

(c) R

(d) None of the above

where R is the set of all real numbers and Z is the set of all integers.

[Here |x| represents greatest integer function]

42. The function $f(x) = x(1 - x)$ defined on $(0, 1)$

(a) has a minimum value of $1/4$

(b) has a maximum value of $1/2$

(c) has a maximum value of $1/4$

(d) has neither maximum nor minimum

43. The equation $\sec 0 = \lim\limits_{y \to \alpha x} \dfrac{x^3 - y^3}{3xy^2 + 3yx^2}$ is valid if

(a) α is a negative integer

(b) $\alpha = 0\cdot 5$

(c) $\alpha = 1$

(d) $\alpha = 2$

44. For $0 < |x| < \dfrac{\pi}{2}$, the value of $1 - \dfrac{\sin x}{x}$ is

(a) strictly positive

(b) strictly negative

(c) non-negative

(d) 0

45. If $27a + 9b + 3c + d = 0$, then the equation $4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one real root lying between

(a) -1 and 0

(b) 0 and 3

(c) -3 and -1

(d) None of the above

46. If the surface area of a spherical balloon is increasing at the rate of 16 square cm per second when the radius is 40 cm, at what rate is the volume increasing at that moment ?

(a) 320 cubic cm per second

(b) 160 cubic cm per second

(c) 150 cubic cm per second

(d) 120 cubic cm per second

47. What is the degree of the differential equation $\dfrac{d^2y}{dx^2} = y^2\sqrt{y}$?

(a) 1

(b) 2

(c) 3

(d) 4

48. The general solution of the differential equation $y\dfrac{dy}{dx} = \left(\dfrac{dy}{dx}\right)^2 x + 1$ is $Ay - A^2x = 1$, where A is the arbitrary constant ($A \neq 0$). Which one of the following solutions of the given differential equation is **not** a particular solution of the equation ?

(a) $y - 4x = 1$

(b) $3y - 9x = 1$

(c) $y + 2x = 0$

(d) $y^2 = 4x$

49. What is the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{2y\left(\sin y^2 + \cos y^2\right) - x}$?

(a) $xy + \cos y^2 - \sin y^2 = c$

(b) $xy + \cos 2y = c$

(c) $xy - y^2 = c$

(d) $xy = c$

where c is an arbitrary constant.

50. After eliminating the arbitrary constant α from $y = \cos(x + \alpha) + 3\sin(x + \alpha)$ to get a differential equation of minimum order as

(a) $y - 3\dfrac{dy}{dx} = 1$

(b) $3y + \dfrac{dy}{dx} = 5$

(c) $y^3 - \left(\dfrac{dy}{dx}\right)^3 = 9$

(d) $y^2 + \left(\dfrac{dy}{dx}\right)^2 = 10$

51. What is $\displaystyle\int_{-1}^{1} \dfrac{x^3}{x^2 + 2|x| + 1}\, dx$ equal to ?

(a) $\ln 2$

(b) $2\ln 2$

(c) 1

(d) 0

52. What is $\displaystyle\int_{\frac{1}{3}}^{\frac{2}{3}} \dfrac{\ln x}{\ln\left(x - x^2\right)}\, dx$ equal to ?

(a) 1/3

(b) 1/6

(c) 1/18

(d) 1/54

53. What is the derivative of $\log_e e^x$?

(a) 2x

(b) e

(c) 1

(d) 2e

54. Let f(x) be a polynomial of degree three satisfying $f(0) = -1$ and $f(1) = 0$. Also '0' is a stationary point of f(x) but f(x) has no extremum value at $x = 0$. What is $\int \frac{f(x)}{x^3 - 1} dx$ equal to ?

(a) $(x^2/2) + c$

(b) $x + c$

(c) $(x^3/3) + c$

(d) None of the above

where c is the constant of integration.

Directions : *For the next **two (02)** questions that follow :*

Given $f(x) = \sin x + \cos x$ and $g(x) = \frac{|x|}{x}$ for $x = 0$ and $g(0) = 2$.

55. What is $\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} g \circ f(x) \, dx$ equal to ?

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) π

56. What is $\int_{\frac{3\pi}{4}}^{\frac{7\pi}{4}} g \circ f(x) \, dx$ equal to ?

(a) 0

(b) $-\frac{\pi}{4}$

(c) $-\frac{3\pi}{4}$

(d) $-\pi$

Directions : *For the next **two (02)** questions that follow :*

Let $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and

$g(x) = f(x - 1) + f(x + 1)$ for all $x \in R$.

57. What is $\int_{-3}^{0} g(x) \, dx$ equal to ?

(a) 0

(b) 1

(c) 2

(d) None of the above

58. What is $\int_{0}^{3} g(x) \, dx$ equal to ?

(a) 0

(b) 1

(c) 2

(d) None of the above

Directions : *For the next **two (02)** questions that follow :*

Let the area enclosed by the curve $y = 1 - x^2$ and above the line $y = a$, where $0 \le a < 1$, be represented by A(a).

59. What is $\frac{A(a)}{A(0)}$ equal to ?

(a) $\frac{1}{2}(1 + 2a)\sqrt{1 - a}$

(b) $\frac{1}{3}a\sqrt{1 - a}$

(c) $\frac{1}{2}(2 + a)\sqrt{1 - a}$

(d) None of the above

60. If $\frac{A(0)}{A(1/2)} = k$, then which one of the following is correct ?

(a) $k = 0$

(b) $0 < k < 0.5$

(c) $0.5 < k < 1$

(d) None of the above

61. Let $f(x) = \begin{cases} x^2 & x \in Z \\ \dfrac{k(x^2-4)}{2-x}, & x \notin Z \end{cases}$ where Z is the set of all integers. Then $f(x)$ is continuous for

   (a) $k = 1$ only
   (b) every real k
   (c) every real k except $k = -1$
   (d) $k = -1$ only

62. What is the area enclosed by the curve $y^2 = -x|x|$ and $x = -1$?

   (a) 1/2 square units
   (b) 1 square unit
   (c) 2 square units
   (d) None of the above

63. If $I_1 = \displaystyle\int_e^{e^2} \dfrac{dx}{\ln x}$ and $I_2 = \displaystyle\int_1^2 \dfrac{e^x \, dx}{x}$ then which one of the following is correct ?

   (a) $2I_1 = I_2$
   (b) $I_1 = 2I_2$
   (c) $I_1 = I_2$
   (d) $I_1 = e\, I_2$

64. If $I_r = \displaystyle\int_0^{\frac{\pi}{4}} \tan^r x \, dx$, then $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6 \ldots$ are in

   (a) AP
   (b) GP
   (c) HP
   (d) None of the above

Directions : *For the next two (02) questions that follow :*

Let $f(x) = \displaystyle\sum_{k=0}^{n} a_k \, |x-1|^k$ where $a_k \in R$.

65. Consider the following statements :

   1. $f(x)$ is a continuous function for all $a_k \in R$.

   2. $f(x)$ is a differentiable function for all $a_k \in R$.

   Which of the statements given above is/are correct ?

   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

66. Consider the following statements :

   1. $f(x)$ is continuous at $x = 1$ provided $a_{2k} = 0$

   2. $f(x)$ is differentiable at $x = 1$ provided $a_{2k+1} = 0$

   Which of the statements given above is/are correct ?

   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

67. What is $\displaystyle\int_{\frac{3\pi}{4}}^{\frac{4\pi}{3}} \frac{\cos x \cdot \sin x}{|\cos x|}\, dx$?

(a) $\dfrac{\sqrt{2}-1}{\sqrt{2}}$

(b) $\dfrac{1+\sqrt{2}}{2}$

(c) $\dfrac{\sqrt{2}-1}{2}$

(d) None of the above

Directions : *For the next* **two (02)** *questions that follow :*

Consider the function $f(x) = |x^2 - 2|$ on the interval $-3 < x < 3$.

68. Consider the following statements :

1. The function attains relative minimum at only one point.

2. The function attains relative maximum at only one point.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

69. Consider the following statements :

1. The absolute maximum value of the function is 2.

2. The absolute minimum value of the function is 0.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

70. If [x] denotes the greatest integer less than or equal to x, then what is
$$\lim_{n\to\infty} \frac{[x]+[2x]+[3x]+\ldots[nx]}{n^2}$$ equal to ?

(a) x

(b) x/2

(c) x/3

(d) 0

Directions : *For the next* **two (02)** *questions that follow :*

A line L is perpendicular to the lines having direction ratios $\langle 3, 2, 1\rangle$ and $\langle 1, 2, 3\rangle$.

71. What is the sum of the direction cosines of the line L ?

(a) 0

(b) 1

(c) 3

(d) None of the above

72. Consider the following statements :

1. The sum of the direction ratios of the line L is equal to sum of the direction cosines of the line L.

2. The angle made by the line L with the y-axis is twice the angle made by the line L with the x-axis.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Directions : *For the next **three** (03) questions that follow :*

Consider the plane containing the line $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{-1}$ and passing through the point $(1, -1, 0)$.

73. What are the direction ratios of the normal to the plane ?

(a) $\langle 1, 1, 1 \rangle$

(b) $\langle 1, -1, 1 \rangle$

(c) $\langle 3, 2, 1 \rangle$

(d) None of the above

74. What is the angle made by the plane with the x-axis ?

(a) $\tan^{-1} \sqrt{2}$

(b) $\cot^{-1} \sqrt{2}$

(c) $\dfrac{\pi}{6}$

(d) None of the above

75. What is the equation of the plane ?

(a) $x + y - z = 0$

(b) $-3x + 2y - z + 6 = 0$

(c) $x + y + z = 0$

(d) None of the above

76. The equation of the plane which equally separates the planes $2x - 2y + z + 3 = 0$, $4x - 4y + 2z + 5 = 0$ is

(a) $8x - 8y + 4z + 13 = 0$

(b) $8x - 8y + 4z + 11 = 0$

(c) $6x - 6y + 3z + 8 = 0$

(d) $6x + 6y + 3z + 8 = 0$

77. The three planes $2x + 3y - z = 2$, $3x + 3y + z = 4$, $x - y + 2z = 5$ intersect

(a) at a point

(b) in a line

(c) at three points

(d) None of the above

Directions : *For the next **two** (02) questions that follow :*

Consider the straight lines $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$ and $x = y - 1 = z$.

78. The lines will be perpendicular as well as coplanar if

(a) $a = b = c$

(b) $-2a = b = -2c$

(c) $2a = 2b = c$

(d) $a = -2b = -2c$

79. The direction cosines of the line $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$ (under the condition given in the previous question) are

(a) $\left\langle \dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right\rangle$

(b) $\left\langle \dfrac{1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right\rangle$

(c) $\left\langle \dfrac{-1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right\rangle$

(d) None of the above

The vectors $x\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + y\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + z\hat{k}$ are coplanar where $x \neq 1$, $y \neq 1$, $z \neq 1$.

80. What is the value of $\dfrac{1}{1-x} + \dfrac{1}{1-y} + \dfrac{1}{1-z}$?

(a) 0

(b) −1

(c) 3

(d) 1

81. What is the value of $xyz - x - y - z$?

(a) 0

(b) 1

(c) 2

(d) −2

82. $\vec{a}, \vec{b}, \vec{c}$ are three non-collinear vectors such that $\vec{a} + \vec{b}$ is parallel to $\vec{c}$ and $\vec{a} + \vec{c}$ is parallel to $\vec{b}$. Then which one of the following is correct ?

(a) $\vec{a} + \vec{b} = \vec{c}$

b $\vec{b} + \vec{c} = \vec{a}$

(c) $\vec{a} + \vec{c} = \vec{b}$

(d) $\vec{a}, \vec{b}, \vec{c}$ taken in order form the sides of a triangle

$\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{\vec{b}}{2}$ where $\vec{b}$ and $\vec{c}$ are non-parallel.

83. What angle does $\vec{a}$ make with $\vec{b}$?

(a) $\dfrac{\pi}{6}$

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{2}$

84. What angle does $\vec{a}$ make with $\vec{c}$?

(a) $\dfrac{\pi}{6}$

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{2}$

85. If $\vec{a}$ and $\vec{c}$ are perpendicular vectors, then for any vector $\vec{b}$ the vectors $\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c}$ are

(a) perpendicular

(b) parallel

(c) each equal to zero vector

(d) parallel respectively to $\vec{a}$ and $\vec{c}$

86. Standard deviation is independent of

(a) change of scale and origin

(b) change of scale but not origin

(c) change of origin but not scale

(d) neither change of scale nor origin

87. X and Y are two related variables. The two regression equations are given by

$2x - y - 20 = 0$ and $2y - x + 4 = 0$.

If $\sigma_y = 1/4$, then what is σ_x equal to ?

(a) 1

(b) 1/2

(c) 1/4

(d) 4

88. The price of petrol per litre during the four quarters (each quarter consists of three months) of a year is ₹ w, ₹ x, ₹ y and ₹ z respectively. A person spends ₹ 200 per month on petrol every month. The average price at which he purchases petrol during the year, in rupees, is

(a) $\dfrac{w + x + y + z}{4}$

(b) $\sqrt{\dfrac{w + x + y + z}{\dfrac{1}{w} + \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}}}$

(c) $\sqrt[4]{wxyz}$

(d) $\dfrac{4wxyz}{wxy + xyz + yzw + zwx}$

89. Following table summarizes the mean and standard deviation of rainfall in three stations A, B and C observed over a period of one month :

Station	A	B	C
Mean rainfall in mm	32	18	44
Standard deviation	6	8	10

In which station/stations is the rainfall consistent ?

(a) A

(b) B

(c) C

(d) A as well as B

90. The average weight of 9 men is x kg. After another man joins the group, the average increases by 5%. Still another man joins and average returns to the old level of x kg. Which one of the following is true ?

(a) The 10th and 11th men weigh the same.

(b) The 10th man weighs half as much as the 11th man.

(c) The 10th man weighs twice as much as the 11th man.

(d) None of the above

91. A point is selected at random from the interior of a circle. The probability that the point is closer to the centre than the boundary of the circle is

(a) 3/4

(b) 1/2

(c) 1/4

(d) 1/8

92. If the letters of the word 'ATTEMPT' are written down at random, the probability that all the T's are consecutive is

(a) 1/42

(b) 6/7

(c) 1/7

(d) 2/7

93. A football match is played from 4 pm to 6 pm. A boy arrives to see the match (not before the match starts). The probability that he will miss the only goal of the match which takes place at the 15th minute of the match is equal to

(a) 3/4

(b) 1/4

(c) 7/8

(d) 1/8

94. Consider the sample space S = {1, 2, 3, ...} of the experiment of tossing a coin until a head appears. Let n denote the number of times the coin is tossed. A probability space is obtained by setting p(1) = 1/2, p(2) = 1/4, p(3) = 1/8 ... .

If an event is defined as A = {n is even}, then what is p(A) equal to ?

(a) 2/3

(b) 2/9

(c) 1/3

(d) 1/9

95. If the sum of mean and variance of a Binomial distribution is 4·8 for five trials, then the distribution is

(a) $\left(\dfrac{1}{3} + \dfrac{2}{3}\right)^5$

(b) $\left(\dfrac{1}{4} + \dfrac{3}{4}\right)^5$

(c) $\left(\dfrac{1}{5} + \dfrac{4}{5}\right)^5$

(d) None of the above

96. Nine numbers are successively drawn from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} replacing the number drawn every time before the next draw. Let X denote the number of 1's in the 9 draws and p_r be the probability that $X = r$ ($r = 0, 1, 2, 3, \dots 9$).

Consider the following statements :

1. $p_0 = p_1$

2. $p_1 = (0.9)^9$

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

97. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, then the probability of getting two heads is

(a) $\dfrac{15}{2^8}$

(b) $\dfrac{2}{15}$

(c) $\dfrac{15}{2^{13}}$

(d) $\dfrac{2}{31}$

98. In a certain college, 25% of the students failed Mathematics, 15% failed Chemistry and 10% failed both Mathematics and Chemistry. A student is selected at random. If the student failed Chemistry, what is the probability that the student failed Mathematics also ?

(a) 2/9

(b) 2/3

(c) 1/3

(d) 1/9

99. From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is

(a) 1/2

(b) 1/3

(c) 2/3

(d) 2/9

100. In a lot of screws, 55% are produced by machine A and the rest by machine B. Machine A produces 2% defective screws and machine B produces 3% defective screws. The probability that a defective screw randomly found from the lot is manufactured by machine B is

(a) 3/5

(b) 29/51

(c) 14/25

(d) 27/49

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