INDIAN NATIONAL MATHEMATICS OLYMPIAD 1987

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Time 3 hours]　　　　**INMO 1987**　　　　[Max Marks 100

Attempt all questions.

Q.1　Given m and n as relatively prime positive integers greater than one, show that $\log_{10}m/\log_{10}n$ is not a rational number.

Q.2　Determine the largest number in the infinite sequence

$$1, \sqrt[2]{2}, \sqrt[3]{3}, \sqrt[4]{4}, \dots \sqrt[n]{n} \dots$$

Q.3　Let T be the set of all triplets (a, b, c) of integers such that $1 \leq a \angle b \angle c \leq 6$ For each triplet (a,b,c) in T, take number axbxc Add all these numbers corresponding to all the triplets in T. Prove that the answer is divisible by 7.

Q.4　If x, y, z, and n are natural numbers, and $n \geq z$ then prove that the relation $x^n + y^n = z^n$ does not hold.

Q.5　Find a finite sequence of 16 numbers such that:

[a] it reads same from left to right as from right to left.

[b] the sum of any 7 consecutive terms is – 1,

[c] the sum of any 11 consecutive terms is + 1.

Q.6 Prove that if coefficients of the quadratic equation $ax^2+bx+c=0$ are odd integers, then the roots of the equation cannot be rational numbers.

Q.7 Construct the Δ ABC, given h_a, h_b (the altitudes from A and B) and m_a, the median from the vertex A.

Q. 8 Three congruent circles have a common point 0 and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the in-centre and the circum-centre of the triangle and the common point 0 are collinear.

Q. 9 Prove that any triangle having two equal internal angle bisectors (each measured from a vertex to the opposite side) is isosceles.

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