

INDIAN NATIONAL MATH OLYMPIAD 1986

Time : 3 hours]

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[Max Marks 100

Attempt all questions.

Q.1 A person who left home between 4 p.m. and 5 p.m. returned between 5 p.m. and 6 p.m. and found that the hands of his watch had exactly exchanged place, when did he go out ?

Q.2 Solve.

$$\log_2 x + \log_4 y + \log_4 z = 2$$

$$\log_3 y + \log_9 z + \log_9 x = 2$$

$$\log_4 z + \log_{16} x + \log_{16} y = 2$$

Q.3 Two circles with radii a and b respectively touch each other externally. Let c be the radius of a circle that touches these two circles as well as a common tangent to the two circles. Prove that

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

Q.4 Find the least natural number whose last digit is 7 such that it becomes 5 times larger when this last digit is carried to the beginning of the number.

Q.5 If $P(x)$ is a polynomial with integer coefficients and a, b, c , three distinct integers, then show that it is impossible to have $P(a) = b$, $P(b) = c$, $P(c) = a$.

Q.6 Construct a quadrilateral which is not a parallelogram, in which a pair of opposite angles and a pair of opposite sides are equal.

Q.7 If a, b, x, y are integers greater than 1 such that a and b have no common factor except 1 and $x^a = y^b$ show that $x = n^b, y = n^a$ for some integer n greater than 1.

Q.8 Suppose A_1, A_6 are six sets each with four elements and $B_1 \dots B_n$ are n sets each with two elements, Let $S = A_1 \cup A_2 \cup \dots \cup A_6 = B_1 \cup \dots \cup B_n$. Given that each element of S belongs to exactly four of the A 's and to exactly three of the B 's, find n .

Q.9 Show that among all quadrilaterals of a given perimeter the square has the largest area.
