11th KVS Maths Olympiad Contest – 2008

Time : 3 Hours M.M. : 100

NOTE: Attempt all questions. No electronic gadget is allowed during the examination.

1) Find the value of $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots\ldots - 98^2 + 99^2$

2) Find the smallest multiple of '15' such that each digit of the multiple is either '0' or '8'.

3) At the end of year 2002. Ram was half as old as his grandfather. The sum of years in which they were born is 3854. What is the age of Ram at the end of year 2003?

4) Find the area of the largest square, which can be inscribed in a right angle triangle with legs '4' and '8' units.

5) In a Triangle the length of an altitude is 4 units and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.

6) A number 'X' leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of 'X'?

7) A sports meet was organized for four days. On each day, half of existing total medals and one more medal was awarded. Find the number of medals awarded on each day.

8) Let ΔABC be isosceles with $\angle ABC = \angle ACB = 78^0$. Let D and E be the points on sides AB and AC respectively such that $\angle BCD = 24^0$ and $\angle CBE = 51^0$. Find the angle $\angle BED$ and justify your result.

9) If α, β and γ are the roots of the equation.

$(x - a)(x - b)(x - c) + 1 = 0.$

Then show that a, b and c are the roots of the equation

$(\alpha - x)(\beta - x)(\gamma - x) + 1 = 0.$

10) A 4 x 4 x 4 wooden cube is painted so that one pair of opposite faces is blue, one pair green and one pair red. The cube is now sliced into 64 cubes of side 1 unit each.

(i) How many of the smaller cubes have no painted face?

(ii) How many of the smaller cubes have exactly one painted face?

(iii) How many of the smaller cubes have exactly two painted faces?

(iv) How many of the smaller cubes have exactly three painted faces?

(v) How many of the smaller cubes have exactly one face painted blue and one face painted green ?

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Solutions

Q.1.Find the value of $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots - 98^2 + 99^2$

Solution : Let

$S = 1^2 - 2^2 - 3^2 - 4^2 + \ldots\ldots\ldots - 98^2 + 99^2$

$= 1^2 (-2^2+3^2) (-4^2+5^2) - (6^2+7^2) \ldots\ldots\ldots\ldots$

$- \quad 98^2 + 99^2$

$= 1^2 + (3+2)(3-2) + (4+5)(5-4)$

$+ (6+7)(7-6) + \ldots\ldots\ldots + (99-98)(99+98)$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + \ldots\ldots\ldots + 98 + 99$

Further,

$S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + \ldots\ldots\ldots + 98 + 99$

$+S = 99 + 98 + 97 + 96 + 95 + 94 + 93 + \ldots\ldots\ldots + 2 + 1$

$\Rightarrow 2S = 100+100+100+100+100+100+100 \ldots\ldots + 100 + 100$

$= 99 \times 100$

$\Rightarrow S = 50 \times 99$

= 4950

Q.2. Find the smallest multiple of ‘15’ such that each digit of the multiple is either ‘0’ or ‘8’.

Solution:

Smallest multiple of 15, such that each digit of the multiple in either 0 or 8 are

Two & Three digit nos	Four digit and Five digit nos
80	8000
880	8008
808	8080
800	8800
	8880
	80888
	80888
	88088

So only possibility for multiple of 15 i.e. divisible by 5

is last digit is 0 i.e.

(i) 2 digits 80

(ii) 3 digits 880, 800

(iii) 4 digit s 8000, 8800, 8880, 8080

(iv) 5 digit 88880 80000

88800 88000

88080

As 15 = 5x3

So the number should be divisible by 3 the sum of digit should be divisible by 3.

Hence let us analyze the sum of digits in (i), (ii), (iii) and (iv),

(i) 2 digit : not possible

(ii) 3 digit : not possible

(iii) 4 digit : with 888

sum in 8+8+8 = 24

that is divisible by 3

But last digit should be 0 and it should contain three numbers of 8.

i. e. 8880

Q. 3 At the end of year 2002. Ram was half as old as his grandfather. The sum of years in which they were born is 3854. What is the age of Ram at the end of year 2003?

Let age of Ram at the end of 2002 = x

so age of his grand father = $2x$

so, Ram was born in $(2002 - x)$

Ram's grand father was born in $(2002-2x)$

from question : $2002 – x + 2002 – 2x = 3854$

$\Rightarrow \quad -3x = -150$

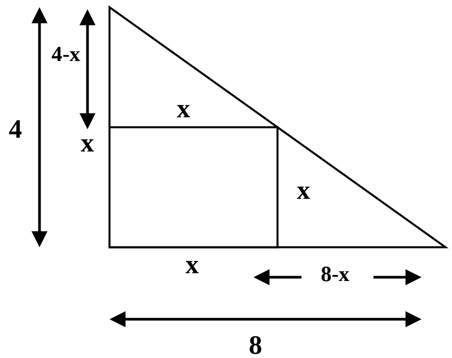
$\Rightarrow \quad x = 50$

So, the age of Ram at the end of 2003 in **51**.

Q. 4 Find the area of the largest square, which can be inscribed in a right angle triangle with legs ‘4’ and ‘8’ units.

Solution:

Let the side of square is x as in figure.



From property of similar triangle

$$\frac{4-x}{x} = \frac{x}{8-x}$$

$\Rightarrow$ $(4-x)(8-x) = x^2$

$\Rightarrow$ $32 - 4x - 8x + x^2 = x^2$

$\Rightarrow$ $32 = 12x$

$\Rightarrow$ $x = \frac{8}{3} = 2.67$

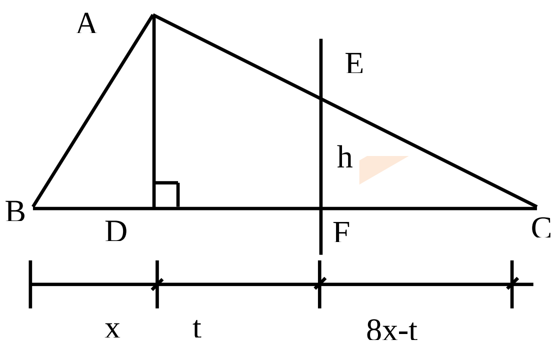
so Area of largest square in $\frac{64}{9}$ sq units

$= 7\frac{1}{9}$ sq units.

Q.5. In a Triangle the length of an altitude is 4 units and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.

Solution:

Let the triangle is ABC an in figure



AD = 4 unit

BD = x

DC = 8 x

Let EF = h and

DF = t

So Area ABD = $\frac{1}{2}$ X 4 X x = 2x

Area ABC = $\frac{1}{2}$ X 9x X 4 = 18x

Let EF bisect the area of ABC, and EF $\parallel$ AD

So,

area EFC = 9x

$\frac{1}{2}$

= X(8x-t) X h

$\Rightarrow h = \dfrac{18x}{8x - t}$ …………(1)

The area of ADFE

$= \dfrac{1}{2}$ X(AD+EF) X t

$= \dfrac{1}{2}$ X (AD + EF) X t

$= (9x – 2x)$ [i.e. half area – area ABD of triangle]

$= 7x$

$\Rightarrow$ 7x= $\dfrac{1}{2}$ X (4+h) X t

$t = \dfrac{14x}{h+4}$ ..…………..(2)

putting t form (ii) in (i)

18x = (8x - t)

$= \left(8x - \dfrac{14x}{h+4}\right) h$

$\Rightarrow 18 = \left(8x - \dfrac{14x}{h+4}\right) h$

$\Rightarrow 18 = \left(\dfrac{8\,(h = 4) – 14}{h + 4}\right) h$

$\Rightarrow$ 18 (h + 4) = [8 (h+4) - 14] h

$\Rightarrow$ 18 h + 72 = [8h + 32 - 14] h

= [8 h + 18] h

= $8 h^2 + 18 h$

$\Rightarrow$ $8 h^2 = 72$

$\Rightarrow$ $h^2 = \frac{72}{8}$

= 9

h = 3 so, height = 3 units Ans.

Q.6 A number 'X' leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of 'X'?

Solution:

Let p, q, r and s be any number from the question, if r in remainder.

5814 = p X + r …………………. (i)

5430 = q X + r …………………. (ii)

5958 = s X + r ………………….. (iii)

from (i) & (ii)

384 = (p-q) X

from (ii) & (iii)

5430 – 5958 = (q - s) X

$\Rightarrow$ 528 = (s - q) X

from (iii) & (i)

5814 – 5958 = (p - s) X

$\Rightarrow$ 144 = (s - p) X

so we get three equation

384 = (p - q) X

528 = (s - q) X

144 = (s - p) X

$\Rightarrow$ (p - q) X = 2x2x2x2x2x2x2x3

(s - q) X = 2x2x2x2x x3x 11

(s - q) X = 2x2x2x2x x3x x3

So the HCF of these three numbers

= 2x2x2x2x3

= 48

So the required largest number is 48

Check:

48 x 121 = 5808 then + 6 = 5814

48 x 113 = 5424 then + 6 = 5430

48 x 124 = 5952 than + 6 = 5958

Q.7. A sports meet was organized for four days. On each day, half of existing total medals and one more medal was awarded. Find the number of medals awarded on each day.

Solution:

Let total medals = m

Medals distributed on

1st day = $\frac{m}{2} + 1 = \frac{m + 2}{2}$

Remaining medals for 2nd day

= m – [medals distributed on 1st day]

$= m - \left(\frac{m - 2}{2}\right)$

$= \frac{m - 2}{2}$

So, medals distributed on 2nd day

$= \left(\frac{m + 2}{4}\right) + 1$

$= \left(\frac{m + 2}{4}\right)$

Remaining medals for 3rd day

= m – [medals distributed or 1st + 2nd day]

$= m - \left(\frac{m + 2}{2} + \frac{m + 2}{4}\right)$

$$= \frac{m - 6}{4}$$

So, medals distributed on 3rd day

$$= \frac{m - 6}{8} + 1$$

$$= \frac{m + 2}{8}$$

Remaining medals for 4th day

= m – [medals distributed on 1st + 2nd + 3rd day]

$$= m - \left(\frac{m + 2}{2} + \frac{m + 2}{4} + \frac{m + 2}{8} \right)$$

$$= \frac{m + 14}{8}$$

So, medals distributed on 4th day

$$= \frac{m - 14}{16} + 1$$

$$= \frac{m + 2}{16}$$

So Final remaining medals after distribution on 4^{th} day

= m – [medals distributed on $1^{st}+2^{nd} + 3^{rd} + 4^{th}$ day]

$$= m - \left[\frac{m+2}{2} + \frac{m+2}{4} + \frac{m+2}{8} + \frac{m+2}{16}\right]$$

$$= m - \frac{15m+30}{16}$$

But sport end & hence methods remained = 0

$\Rightarrow$ 16 m = 15m + 30

$\Rightarrow$ m = 30

So day wise medal distribution

Day 1 = 16

Day 2 = 8

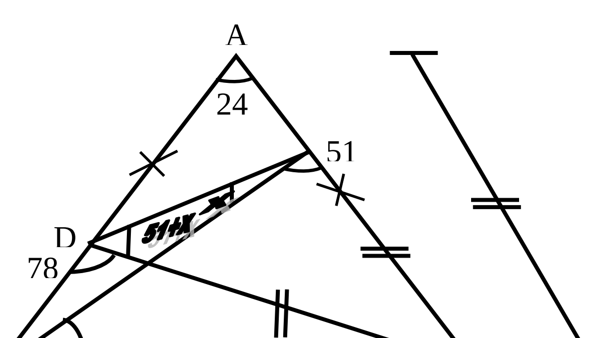
Day 3 = 4

Day 4 = 2

11) Total = 30

Q.8. Let ΔABC be isosceles with ∠ABC = ∠ACB = 78^0. Let D and E be the points on sides AB and AC respectively such that ∠BCD = 24^0 and ∠CBE = 51^0. Find the angle ∠BED and justify your result.

Solution:



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From question AB = AC

So $\angle BDC = 180 - (24+78)$

$= 78^0$

and $\angle DBC = 78^0$

So BC = DC

Further, $\angle BEC = 180 - (78+51)$

$= 51$

So BC = EC = DC

Let $\angle DEB = x$,

So in isosceles CED,

$\angle CDE = \angle CED$

$= 51+x$

Let BE and CD meet at F

So $\angle DFE = 105^0$

Hence in Δ DFE

$51+x + 105 + x = 180^0$

$\Rightarrow \quad 2x = 24$

$\Rightarrow \quad x = 12^0$

Justification

$\angle CDE = 51+12$

$= 63$

Hence $\angle EDA = 180 - (78+63)$

$= 39$

Hence , $\angle AED = 180 - (24+39)$

$= 117^0$

so $117 + 12 + 51 = 180^0$

Hence justified