# KVS Junior Mathematics Olympiad (JMO) – 2005

M.M. 100 Time : 3 hours

Note : (i) Attempt all questions. Each question carries ten marks.

(ii) Please check that there are two printed pages and ten Questions in the question paper.

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1. Fill in the blanks:

(a) If four times the reciprocal of the circumference of a circle equals the diameter of the circle, then the area of the circle is …………..

(b) If $1-\frac{4}{x}+\frac{4}{x^2}=0$ then $\frac{2}{x}$ equals………

(c) If a=1000, b=100, c=10, and d=1, then

(a+b+c-d) + (a+b-c+d) + (a-b+c+d)+(-a+b+c+d) is equal to …….

(d) When the base of a triangle is increased by 10% and the altitude to the base is decreased by 10%, the change in area is ……….

(e) If the sum of two numbers is 1, and their product is 1, then the sum of their cubes is ……………

2. (a) If x = $\left(\log_8^2\right)^{\log_2^8}$ find the value of $\log_3 x$.

(b) If $\frac{4^x}{2^{x+y}}=8$ and $\frac{9^{x+y}}{3^{5y}}=243$ find the value of x-y.

3. (a) Find the number of digits in the number $2^{2005} \times 5^{2000}$ when

written in full.

(b) Find the remainder when $2^{2005}$ is divided by 13.

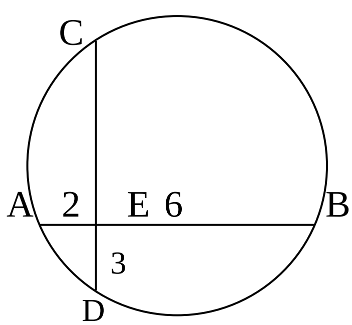
4. (a) A polynomial p (x) leaves a remainder three when divided by x – 1 and a remainder five when divided by x-3. Find the remainder when p(x) is divided by (x-1) (x-3).

(b) Find two numbers, both lying between 60 and 70, each of which is exactly divides $2^{43}$-1.

5. In triangle ABC the medians AM and CN to the sides BC and AB, respectively intersect in the point O.P is the mid-point of side AC, and MP intersects CN in Q. If the area of triangle OMQ is 24 $cm^2$, find the area of triangle ABC.

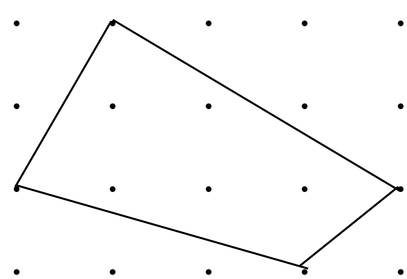
6. The base of a pyramid is an equilateral triangle of side length 6 cm. The other edges of the pyramid are each of length $\sqrt{15}$ cm. Find the volume of the pyramid.

7. Chords AB and CD of a circle (see figure) intersect at E and are perpendicular to each other segments AE. EB and ED are of lengths 2cm, 6cm and 3cm respectively. Find the length of the diameter of the circle.



8. Three men A, B and C working together, do a job in 6 hours less time than A alone, in 1 hour less time than B alone, and in one half the time needed by C when working alone. How many hours will be needed by A and B working together, to do the job ?

9. Pegs are put on a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure forming a quadrilateral. Find the area of the quadrilateral in square units.



10. The odd positive integers 1, 3, 5, 7 ….. are arranged in five columns continuing with the pattern shown on the right. Counting from the left, in which column (I, II, III, IV or V) does the number 2005 appear ? (Justify your answer)

| I | II | III | IV | V |
|---|---|---|---|---|
| | 1 | 3 | 5 | 7 |
| 15 | 13 | 11 | 9 | |
| | 17 | 19 | 21 | 23 |
| 31 | 29 | 27 | 25 | |
| | 33 | 35 | 37 | 39 |
| 47 | 45 | 43 | 41 | |
| | 49 | 51 | 53 | 55 |
| . | . | . | . | |
| | . | . | . | . |
| . | . | . | . | |

# KV JMO 2005 SOLUTIONS

Q1.

(a) Circumference = $2\pi r$

Diameter = $2r$

$\therefore\ 4x\dfrac{1}{2\pi r} = 2r$

$\Rightarrow 1 = \pi\ r\ x\ r$

$\Rightarrow 1 = \pi\ r^2$

$\Rightarrow \pi r^2 = 1$

$\therefore$ Area = $\pi\ r^2$

= 1 sq. units

(b) $1-\dfrac{4}{x}+\dfrac{4}{x^2}=0$

$\Rightarrow 1-2.(1)x\left(\dfrac{2}{x}\right)+\left(\dfrac{2}{x}\right)^2=0$

$\Rightarrow \left(1-\dfrac{2}{x}\right)^2=0$

$\Rightarrow 1-\dfrac{2}{x}=0$

$\Rightarrow 1=\dfrac{2}{x}$

$\Rightarrow$ x = 2

$\therefore \ \frac{2}{x} = \frac{2}{2} = 1$

(c) a = 1000, b = 100, c = 10, d=1

(a+b+c-d) + (a+b-c+d) + (a-b+c+d)+(-a+b+c+d)

= 2 (a+b+c+d)

=2(1000+100+0+1)

= 2 x 1111

= 2222 Ans

Q1. (d) If Base = 6

Attitude = a

$\therefore$ Area = $\frac{1}{2}$ ab

Now, New Area = $\frac{1}{2}$ x($\left(\frac{90}{100}a\right)$) x $\left(\frac{110}{100}b\right)$

= $\frac{99}{200}ab$

Decrease in Area = $\frac{1}{2}$ ab - $\frac{99}{200}ab$

= $\frac{100ab - 99ab}{200}$

= $\frac{1}{200}ab$

$\therefore$ Decrease Percentage = $\dfrac{\frac{1}{200}ab}{\frac{1}{2}ab} x100$

$\dfrac{1}{100} x100$

= 1% decrease.

(e)Let, the two number be x and y

$\therefore$ x + y = 1

$\therefore$ x y = 1

$\therefore$ $(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 12 = x^2 + y^2 + 2 \text{ x } 1$

$\Rightarrow x^2+y^2 = 1-2$

= -1

$\therefore$ $x^3 + y^3 = (x + y) \cdot (x^2 + y^2 - xy)$

$\Rightarrow (1) \cdot (-1 - 1)$

= -2

Q2(a) $x = \left(\log_8^2\right)^{\log_2^8}$

$\text{Log}_3 x = ?$

Let, $\log_8{2} = m$

$\Rightarrow m = \log_8{2}$

$\Rightarrow 8^m = 2$

$\Rightarrow 8^m = 8^{1/3}$

$\Rightarrow m = \frac{1}{3}$

let, $\log_2{8} = n$

$\Rightarrow n = \log_2{8}$

$\Rightarrow 2^n = 8$

$\Rightarrow 2^n = 2^3$

$\Rightarrow n = 3$

$\therefore\ x = \left(\log_8^2\right)^{\log_2^8}$

$(m)^n$

$= \left(\frac{1}{3}\right)^3$

$= \frac{1}{27}$

Let, $p = \log_3 x$

$\Rightarrow p = \log_3 \frac{1}{27}$

$\Rightarrow 3^p = \frac{1}{27}$

$\Rightarrow 3^p = \left(\frac{1}{3}\right)^3$

$\Rightarrow 3^P = (3)^{-3}$

$\Rightarrow p = -3$

$\therefore \log_3 x = -3$

Q2.(b) $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$

x-y = ?

$$\frac{4^x}{2^{x+y}} = 8$$

$$\Rightarrow \frac{(2^2)^x}{2^{x+y}} = 8$$

$$\Rightarrow \frac{2^{2x}}{2^{x+y}} = 8$$

$$\Rightarrow 2^{2x-x-y} = 8$$

$$\Rightarrow 2^{x-y} = (2)^3$$

$$\Rightarrow x - y = 3$$

$\therefore$ x-y= 3

3(a) $2^{2005} \times 5^{2000}$

$= 2^5 \times 2^{2000} \times 5^{2000}$

$= 2^5 \times (2 \times 5)^{2000}$

$= 2^5 \times 10^{2000}$

$= 32 \times 10^{2000}$

= 320000000 …. (2000 zeros)

/ There are 2002 digits in the above number.

Q 3(b) $2^1 \equiv 2 \pmod{13}$

$2^2 \equiv 4 \pmod{13}$

$2^3 \equiv 8 \pmod{13}$

$2^4 \equiv 16 \pmod{13}$

$\equiv 3 \pmod{13}$

$2^5 \equiv 32 \pmod{13}$

$\equiv 6 \pmod{13}$

$2^6 \equiv 64 \pmod{13}$

$\equiv -1 \pmod{13}$

$\Rightarrow (2^6)^{334} = (-1)^{334} \pmod{13}$

$\Rightarrow 2^{2004} = 1 \pmod{13}$

$\Rightarrow 2^{2005} = 2 \pmod{13}$

$\therefore$ $2^{2005}$ leaves a remainder 2 on division by 13.

4(a)

$\because$ The polynomial gives a remainder 3 on division by x –1.

Let, p (x) = k(x-1) + 3

= kx – k + 3

Now,

$$\begin{array}{r|l} & \quad\quad\quad k \\ x-3 & kx - k + 3 \\ & kx - 3k \\ \hline & \quad\quad 2k + 3 \end{array}$$

$\therefore$ Reminder = 2k + 3

But,

2k + 3 = 5

$\Rightarrow$ 2k = 2

$\Rightarrow$ k = 1

$\therefore$ P(x) = k (x-1) + 3

= 1 (x-1) + 3

=x-1 + 3

= x +2

Now ,

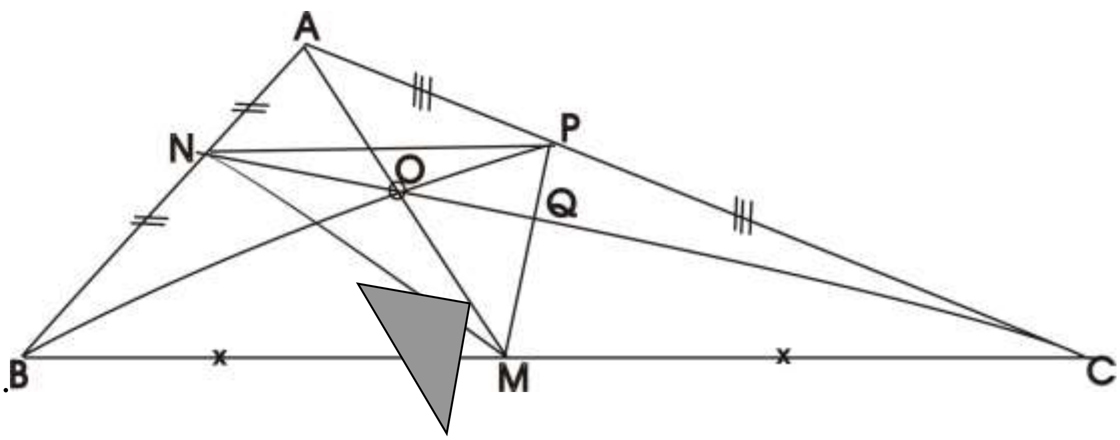
(x-1) (x-3)

= $x^2 - 4x + 3$

Dividing p(x) by $x^2 - 4x + 3$

$$\begin{array}{r|l} & 0 \\ x^2 - 4x + 3 & x + 2 \\ & 0 + 0 \\ \hline & x + 2 \end{array}$$

Hence, the required remainder = x + 2.

Q5.



Given that ar (ΔOMQ) = 24 $cm^2$

So, ar (ΔMOP) = 24 x 24 $cm^2$

= 48 $cm^2$

[Being same height and MQ = QP]

So, ar (ΔBOM) = 2 x ar (ΔMOP)

$= 96\ cm^2$

[Taking BO as base, having same height as ΔMOP on base OP]

So, ar (ΔAOP) = 2 x ar (ΔOMP)

$= 2 \times 48\ cm^2$

$= 96\ cm^2$

Area (ΔBOA) = 2 x ar (ΔAOP)

$= 192\ cm^2$

Area (ΔMPR) = ar (ΔBMP)

= ar (ΔBOM + ΔMOP)

$= 96 + 48\ cm^2$

$= 144\ cm^2$

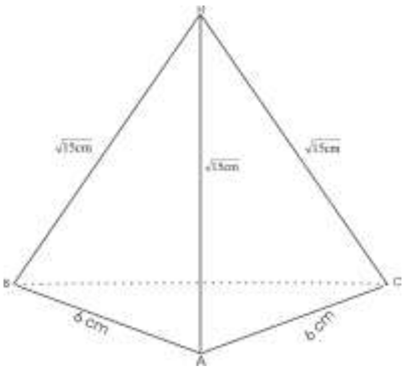
So ar (ΔABC) = ar (ΔMPR) + ar (ΔBPM)

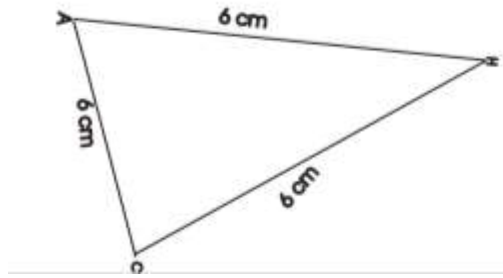
+ ar (ΔAOP) + ar(ΔBOA)

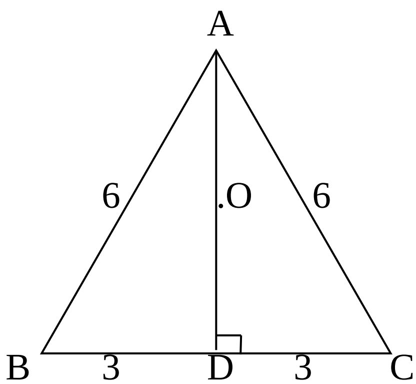
$= 144 + 144 + 96 + 192\ cm^2$

$= 576\ cm^2$

Q6.




In $\Delta ABC$,

$$AD = \sqrt{6^2 - 3^2}$$

$$= \sqrt{36 - 9}$$

$$= \sqrt{27}$$

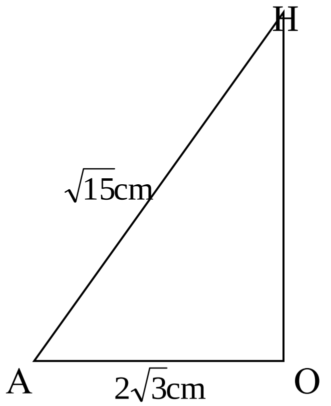
$$= 3\sqrt{3}$$

$$\therefore AO = = \frac{2}{3} \text{ x } AD$$

$$= \frac{2}{3} \text{x} 3\sqrt{3}$$

$$= 3\sqrt{3}\text{cm}$$

Now, In $\Delta AOH$




$\therefore$ OH $= \sqrt{(15)^2 - \left(2\sqrt{3}\right)^2}$

$= \sqrt{15-12}$

$= \sqrt{3}$

ar ($\Delta ABC$) = $\frac{\sqrt{3}}{4} x 6^2$

$= \frac{\sqrt{3}}{4} x 6 x 6$

$= 9\sqrt{3}$

$\therefore$ Volume of the pyramid = $\frac{1}{3}$ (Base) x (Height)

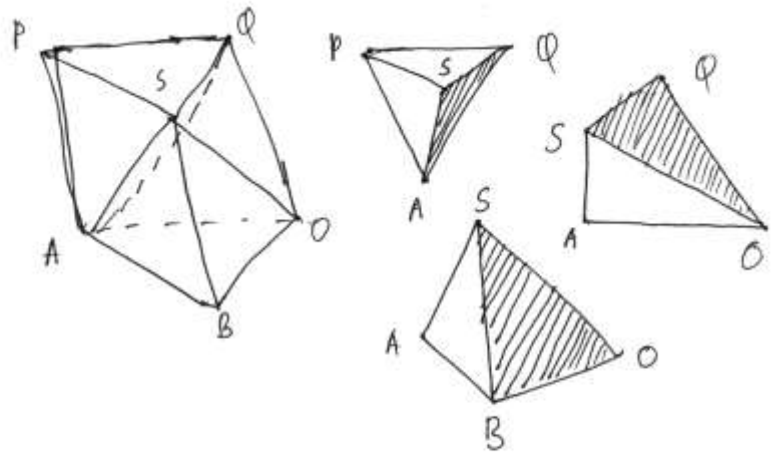
$= \frac{1}{3} x 9\sqrt{3} x \sqrt{3}$

= 9 $cm^3$

PLEASE REFER TO THE NOTE GIVEN TO THIS Q

Note for Q.6

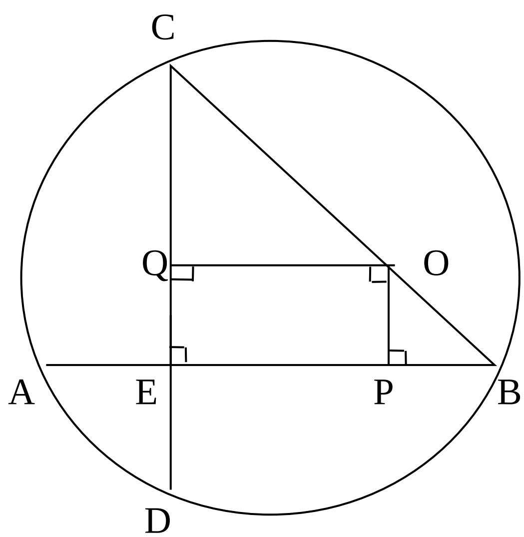
A regular right triangular prism can be divided into three equal and regular right pyramids as shown below :




∴ Volume of pyramid = 1/3 (volume of a prism)

= 1/3 (base area) (height)

Q7.




**GIVEN :**

CD || AB

AE = 2 cm

BE = 6 cm

DE = 3 cm

**CONSTRUCTION :**

Draw OQ⊥CD

And OP⊥AB

**TO FIND**

Diameter of circle = ?

**PROCESS :**

∵ Perpendicular drawn from the center of a circle to the chord, bisect the chord.

∴ AP = PB

and CQ = QD

∴ AP = PB = ½ AB

= ½ (AE + EB)

= ½ (2+6)

= ½ x 8

= 4 cm

In rectangle QOPE,

Let, QE = OP = x cm

∴ CQ = QD

= QE + ED

= x + 3

Also,

BE = 6 cm

⇒ BP + PE = 6 cm

⇒ 4 + PE = 6 cm

⇒ PE = 6 - 4 cm

= 2cm

∴ OQ = PE = 2cm

Now,

In right ΔCOQ,

CQ= x + 3

OQ = 2 cm

OC = radius = r

By pathagoras theorem,

$\therefore \ (x + 3)^2 + 2^2 = r^2$

$\Rightarrow x^2 + 6x + 9 + 4 = r^2$

$\Rightarrow x^2 + 6x + 13 = r^2$ (i)

In right ΔBOP,

OP = x

BP = 4

OB = r

$\therefore \ x^2 + 4^2 = r^2$

$\Rightarrow x^2 + 16 = r^2$ (ii)

Find (i) and (ii)

$x^2 + 6x + 13 = x^2 + 16$

$\Rightarrow 6x = 16 - 13$

$\Rightarrow x = \frac{3}{6} = \frac{1}{2}$

$\therefore\ r^2 = x^2 + 16$

$= \left(\frac{1}{2}\right)^2 + 16$

$= \frac{1}{4} + 16$

$= \frac{1+64}{4}$

$= \frac{65}{4}$

$\Rightarrow r = \sqrt{\frac{65}{4}}$

$= \sqrt{\frac{65}{2}}$

$\therefore$ Diameter = 2r

$= 2 \times \sqrt{\frac{65}{2}}$

$= \sqrt{65}$

$\therefore$ Diameter = $\sqrt{65}$ cm

Q8. Let, A, B and C, together do the job in x hrs.

$\therefore$ A does the job in (x+6) hrs.

$\therefore$ B does the job in (x+1) hrs.

$\therefore$ C does the job in (2x) hrs.

$\therefore$ Amount of job done by A in 1 hr. = $\frac{1}{x+6}$

$\therefore$ Amount of job done by B in 1 hr. = $\frac{1}{x+1}$

$\therefore$ Amount of job done by C does in 1 hr. = $\frac{1}{2x}$

According to the question,

$$\frac{1}{x+6} + \frac{1}{x+1} + \frac{1}{2x} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{x+6} + \frac{1}{2x} = \frac{1}{x} - \frac{1}{x+1}$$

$$\Rightarrow \frac{2x+x+6}{2x^2+12x} = \frac{x+1-x}{x^2+x}$$

$$\Rightarrow \frac{3x+6}{2x^2+12x} = \frac{1}{x^2+x}$$

$$\Rightarrow \frac{3x+6}{2x+12} = \frac{1}{x+1}$$

$\Rightarrow (3x + 6)(x+1) = 2x + 12$

$\Rightarrow 3x^2 + 3x + 6x + 6 = 2x + 12$

$\Rightarrow 3x^2 + 9x + 6 - 12 - 2x = 0$

$\Rightarrow 3x^2 + 7x - 6 = 0$

$\Rightarrow 3x^2 + 9x - 2x - 6 = 0$

$\Rightarrow$ 3x (x+3) – 2 (x+3) = 0

$\Rightarrow$ (3x-2) (x+3) = 0

$\Rightarrow$ x=-3 or $\frac{2}{3}$

But negative value is not possible

$\therefore$ x = $\frac{2}{3}$

$\therefore$ Amount of job done by A is 1 hr = $\dfrac{1}{\frac{2}{3}+6}$

$$= \frac{1}{\frac{2+18}{3}}$$

$$= \frac{3}{20}$$

$\therefore$ Amount of job done by B in hr $= \dfrac{1}{\frac{2}{3}+1}$

$$= \frac{1}{\frac{2+3}{5}}$$

$$= \frac{3}{5}$$

Let, time taken by A and B, together to do the job = y

$$\therefore\ y\left(\frac{3}{20}+\frac{3}{5}\right)=1$$

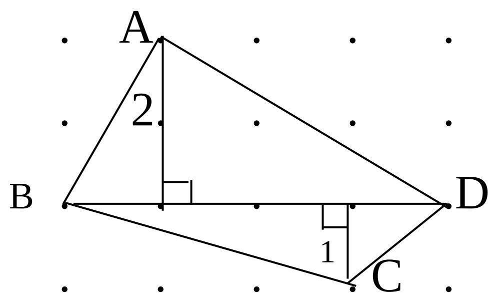
$$\Rightarrow y\left(\frac{3+12}{20}\right)=1$$

$$\Rightarrow y\left(\frac{15}{20}\right)=1$$

$$\Rightarrow y=\frac{20}{15}=\frac{4}{3}=1\frac{1}{3}\text{ hrs.}$$

$\therefore$ A and B, together can do the work is $1\frac{1}{3}$ hrs.

Q9.




$\because$ The dots are 1 units apart, horizontally and vertically

$\therefore$ BD = 1 + 1 + 1 + 1 = 4 units

$\therefore$ Height of $\Delta ABD$ = 1 + 1 = 2 units

$\therefore$ Height of $\Delta BCD$ = 1 unit

$\therefore$ ar (ΔABD) = $\frac{1}{2}$x4x2

= 4 sq. units

$\therefore$ ar (ΔABD) = $\frac{1}{2}$x4x1

= 2 sq. units

$\therefore$ ar (ΔBCD) = ar (ΔABD) + ar (ΔBCD)

= 4 + 2

= 6 sq units.

$\therefore$ Area of the given quadrilateral = 6 sq. units

Q10.

| I | II | III | IV | V |
|---|---|---|---|---|
| | 1 | 3 | 5 | 7 |
| 15 | 13 | 11 | 9 | |
| | 17 | 19 | 21 | 23 |
| 31 | 29 | 27 | 25 | |
| | 33 | 35 | 37 | 39 |
| 47 | 45 | 43 | 41 | |
| . | . | . | . | |
| | . | . | . | . |
| . | . | . | . | |

<u>In column I :</u>

15 = 16 x 1 – 1 = 16k – 1

$31 = 16 \times 2 - 1 = 16k - 1$

$47 = 16 \times 3 - 1 = 16k - 1$

But, $2005 \neq 16K-1$ for any integer K

$\therefore$ 2005 is not in column I.

<u>In column II :-</u>

$1 = 16 \times 0 + 1 = 16k + 1$

$13 = 16 \times 1 - 3 = 16k - 3$

$17 = 16 \times 1 + 1 = 16k + 1$

$29 = 16 \times 2 - 3 = 16k - 3$

But, $2005 \neq 16k + 1$

$2005 \neq 16k + 1$

$\therefore$ 2005 is not a column II,

<u>In column III :-</u>

$3 = 16 \times 0 + 3 = 16k + 3$

$11 = 16 \times 1 - 5 = 16k - 5$

$19 = 16 \times 1 + 3 = 16k + 3$

$27 = 16 \times 2 - 5 = 16k - 5$

But, $2005 \neq 16k + 3$

$2005 \neq 16k - 5$

for any integer k.

$\therefore$ 2005 dosen't occur in column III,

In column IV :-

$5 = 16 \times 0 + 1 = 16k + 5$

$9 = 16 \times 1 - 7 = 16k - 7$

$21 = 16 \times 1 + 5 = 16k + 5$

$25 = 16 \times 2 - 7 = 16k - 7$

But, $2005 \neq 16k - 7$ for any integer k.

But, $16k + 5 = 2005$

For $k = 125$

$\therefore$ 2005 occurs in column IV.