KVS Junior Mathematics Olympiad (JMO) – 2002

M.M. 100 Time : 3 hours

Note : *(i)* *Please check that there are two printed pages and 10 questions in the question paper.*

(ii) *Attempt all questions.*

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1. Fill in the blanks

(a) Yash is carrying 100 hundred – rupee notes, 50 fifty –rupee notes, 20 twenty – rupee notes, 10 ten –rupee notes and 5 five-rupee notes. The total amount of money he is carrying in Rupee, is …………

(b) In a school, the ratio of boys to girls is 4:3 and the ratio of girls to teachers is 8 :1. The ratio of students to teachers is ………

(c) The value of $\left(0.5+\frac{1}{0.5}\right)^2$ is …………

(d) $(123456)^2 + 123456 + 123457$ is the square of ………

(e) The area of square is 25 square centimeters. In perimeter, in centimeters, is …………….

2. (a) How many four digit numbers can be formed using the digits 1,2 only so that each of these digits is used at least once ?

(b) Find the greatest number of four digits which when increased by 1 is exactly divisible by 2, 3, 4, 5, 6 and 7.

3. (a) If $f(x) = ax^7 + bx^5 + cx^3 - 6$, and $f(-9) = 3$, find $f(9)$.

(b) Find the value of

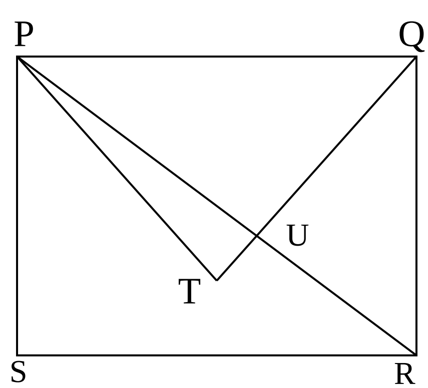
$$\frac{(2002)^3 - (1002)^3 - (1000)^3}{3 \times (1002) \times (1000)}$$

4.(a) If $x > 0$ and $x^4 + \frac{1}{x^4} = 47$, find the value of $x^3 + \frac{1}{x^3}$

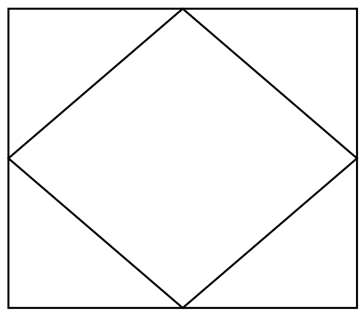
(b) If $8^{2x} = 16^{1-2x}$, find the value of 3^{7x}.

5. A train, after traveling 70 km from a station A towards a station B, develops a fault in the engine at C, and covers the remaining journey to B at $\frac{3}{4}$ of its earlier speed and arrives at B 1 hour and 20 minutes late. If the fault had developed 35 km further on at D, it would have arrived 20 minutes sooner. Find the speed of the train and the distance from A to B.

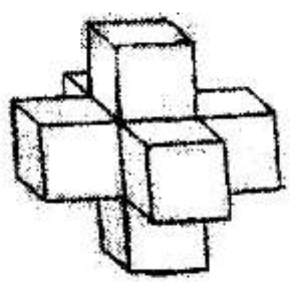
6. The adjoining diagram shows a square PQRS with each side of length 10 cm. Triangle PQT is equilateral. Find the area of the triangle UQR.



A square of side – length 64 cm is given. A second square is obtained by connecting the mid points of the sides of the first square (as shown in the diagram). If the process of forming smaller inner squares by connecting the mid points of the sides of the previous squares is continued, what will be the side-length of the eleventh square, counting the original square as the first square ?



7. Seven cubes of he same size are glued together face to face as shown in the adjoining diagram. What is the surface area, in square centimeters, of the solid if its volume is 448 cubic centimeters ?



8. Anil, Bhavna, Chintoo, Dolly and Eashwar play a game in which each is either a FOX or a RABBIT. FOXES' statements are always false and RABBITS' statements are always true.

Anil says that Bhavna is a RABBIT.

Chintoo says that Dolly is a FOX.

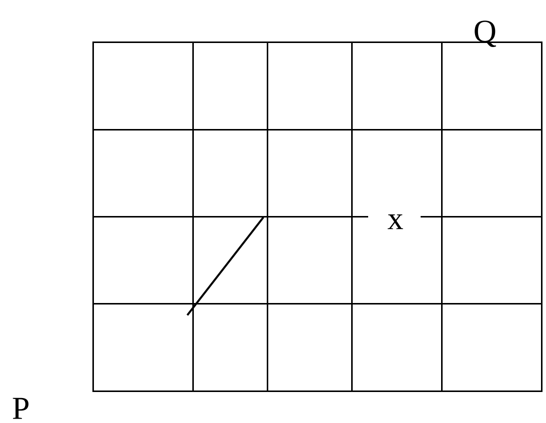
Eashwar says that Anil is not a FOX.

Bhavna says that Chintoo is not a RABBIT.

Dolly says that Eashwar and Anil are different kinds of animals.

How many FOXES are there ? (Justify your answer).

10. The accompanying diagram is a road-plan of a city. All the roads go east-west or north-south, with the exception of one shown. Due to repairs one road is impassable at the point X, of all the possible routes from P to Q, there are several shortest routes. How many such shortest routes are there ?



KV JMO 2002 SOLUTIONS

Q1.

(a) 4025

(b) 56 : 3

(c) 6.25

(d) 123457

(e) 20 cm

Q2.

(a) All the four digit number , which can be formed are :-

1121, 1112, 1122

2221, 2211, 2212

1221, 1212, 1211, 1222

2121, 2112, 2111, 2122

i.e. a total of 14 numbers

(b) First

L.C.M. of 2, 3, 4, 5, 6, and 7

= 420

Now the largest four – digit multiple of 420 is :-

420 x 23 = 9660

$\therefore$ The req. number is = 9660 – 1

= 9659 Ans.

Q3.

(a) $F(x) = ax^7 + bx^5 + cx^3 - 6$

$f(-9) = 3$

$\therefore$ $f(-9) = a(-9)^7 + b(-9)^5 + c(-9)^3 - 6$

$\Rightarrow a(-9)^7 + b(-9)^5 + c(-9)^3 - 6 = 3$

$\Rightarrow -a(9)^7 + b(-9)^5 + -c(9)^3 = 9$

$\Rightarrow -a(9)^7 - b(9)^5 - c(9)^3 = 9$

$\Rightarrow a(9)^7 + b(9)^5 + c(9)^3 = -9$

$\therefore$ $f(9) = a(9)^7 + b(9)^5 + c(9)^3 - 6$

$= -9 - 6$

$= -15$ Ans.

(b) $$\frac{(2002)^3 - (1002)^3 - (1000)^3}{3 \text{ x } (1002) \text{ x } (1000)}$$

$$= \frac{(2002 - 1002)[(2002)^2 + (2002)(1002) + (1002)^2] - (1000)^3}{3 \text{ x } (1002) \text{ x } (1000)}$$

$$= \frac{1000[(2002 - 1002)^2 + 3(2002)(1002)] - (1000)^3}{3 \text{ x } (1002) \text{ x } (1000)}$$

$$= \frac{(1000)^2 + 3(2002)(1002) - (1000)^2}{3 \text{ x } (1002)}$$

$$= \frac{\cancel{3}\,(2002)\cancel{(1002)}}{\cancel{3}\text{ x }\cancel{(1002)}}$$

= 2002 Ans.

Q4.(a) $x^4 + \frac{1}{x^4} = 47$

$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right)^2 + 2.\,(x^2)\left(\frac{1}{x^2}\right)^2 - (\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right)^2 = 47$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 - 2 = 47$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 49$

$\therefore \quad x^2+\frac{1}{x^2} = 7$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 - 2.(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = 7$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 7 + 2$

$\Rightarrow x+\frac{1}{x} = \sqrt{9}$

$= 3$

$$\therefore\ x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-(\not{x})\,x\left(\frac{1}{\not{x}}\right)+\frac{1}{x^2}\right)$$

$$=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$$

$= \quad (3)\ (7 - 1)$

$= \quad 3 \text{ x } 6$

$= \quad 18$ Ans.

(b) $\quad 8^{2x} \quad = \quad 16^{\,1-2x}$

$\Rightarrow \quad 8^{2x} \quad = \quad (8^2)^{1-2x}$

$\Rightarrow \quad (8)^{2x} \quad = \quad (8)^{2-4x}$

$\Rightarrow \quad 2x \quad = \quad 2 - 4x$

$\Rightarrow \quad 2x + 4x \quad = \quad 2$

$\Rightarrow \quad 6x \quad = \quad 2$

$\Rightarrow \quad x \quad = \quad \frac{1}{3}$

$\therefore \quad 3^{7x}$

$= \quad 3^{7 \text{x} \frac{1}{3}}$

$= \quad (3)^{\frac{7}{3}} \quad = \quad (3^3 \text{ x } 3^3 \text{ x } 3^3)^{\frac{1}{3}}$

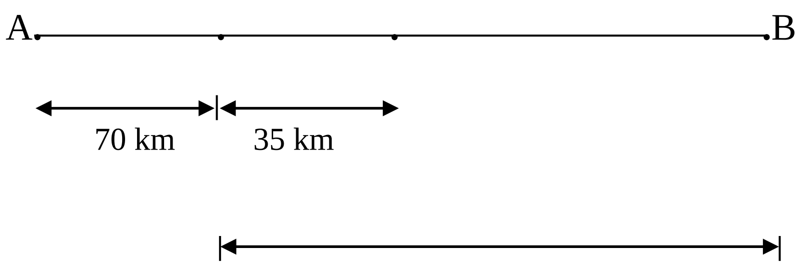
$= \quad 3^{3 \text{x} \frac{1}{3}} \text{ x } 3^{3 \text{x} \frac{1}{3}} \text{ x } 3^{3 \text{x} \frac{1}{3}}$

$$= \quad 3 \text{ x} 3 \text{ x } 3\sqrt{3}$$

$$= \quad 9 \ 3\sqrt{3}$$

$$\therefore \quad 3^{7x} \quad = \quad 9 \ 3\sqrt{3} \ \text{Ans.}$$

5.



Let, the train's speed $= \quad y$

$\therefore$ Total time $= \dfrac{70+x}{y}$

Case I :-

$$\text{Total time} = \frac{70}{y} + \frac{x}{\frac{3}{4}y}$$

$$= \frac{70}{y} + \frac{4x}{3y}$$

$$= \frac{210+4x}{3y}$$

$$\therefore \quad \frac{210+4x}{3y} = \frac{70+x}{y} + 1+\frac{1}{3}$$

$$\Rightarrow \quad \frac{210+4x}{\cancel{3y}} = \frac{210+3x+3y+y}{\cancel{3y}}$$

$\Rightarrow \quad 210 + 4x = 210 + 3x + 4y$

$\Rightarrow \quad x - 4y = 0$ -------------- (i)

Case II :-

$$\text{Total time} = \frac{70+35}{y} + \frac{x-35}{\frac{3}{4}y}$$

$$= \frac{70+35}{y} + \frac{4x-140}{3y}$$

$$= \frac{105 \text{ X } 3 + 4x - 140}{3y}$$

$$= \frac{315-140+4x}{3y}$$

$$= \frac{175+4x}{3y}$$

$$\therefore \quad \frac{175+4x}{3y} = \frac{70+x}{y} + 1$$

$$\Rightarrow \quad \frac{175+4x}{3y} = \frac{70+x+y}{y}$$

$\Rightarrow \quad 175 + 4x = 210 + 3x + 3y$

$\Rightarrow \quad 4x - 3x - 3y = 210 - 175$

$\Rightarrow \quad x - 3y = 45$ (ii)

We have,

(i) $\Rightarrow x - 4y = 0$

(ii) $\Rightarrow x - 3y = 45$

$x - 4y = 0$

$x - 3y = 45$

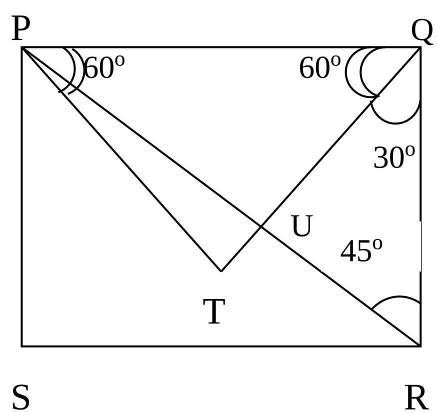
$- \quad + \quad -$

$-y = -45$

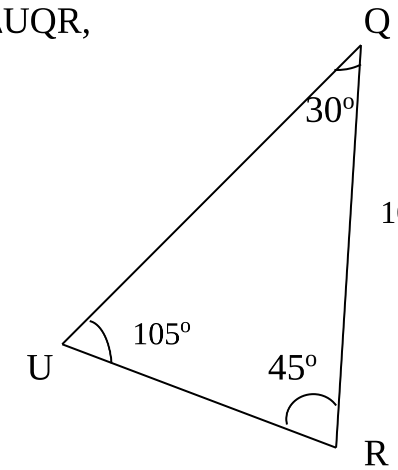
$\Rightarrow y = 45$ km/h

$\therefore$ Speed of train = 45 km / h

Q6.



Considering ΔUQR,



$\text{Sin } \angle QUR = \text{sine } 105^o = \sin(60 + 45)$

$= \sin 60 \cos 45 + \sin 45 \cos 60$

$$= \frac{\sqrt{3}}{2} \text{ x } \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ x } \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

In ΔQUR,

By the law of sine, we have,

$$\frac{UQ}{\sin\angle URQ} = \frac{QR}{\sin\angle QUR} = \frac{UR}{\sin\angle UQR}$$

$$\Rightarrow \frac{UQ}{\sin 45^o} = \frac{10}{\sin 105^o} = \frac{UR}{\sin 30^o}$$

$$\therefore \frac{UQ}{\sin 45} = \frac{10}{\sin 105}$$

$$\Rightarrow UQ = \frac{10.\sin 45}{\sin 105}$$

$$= \frac{10\text{x}\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= 10\text{x}\frac{1}{\sqrt{2}}\text{ x }\frac{2\sqrt{2}}{\sqrt{3}+1}$$

$= \frac{20}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{20(\sqrt{3}-1)}{(\sqrt{3})^2-1^2} = \frac{20(\sqrt{3}-1)}{3-1}$

$= 10(\sqrt{3}-1)$

$\therefore$ UQ = $10(\sqrt{3}-1)$ cm

We know that,

Area of a triangle = ½ bc sin A

$\therefore$ In Δ QUR,

ar (ΔUQR) = ½ x UQ x QR x sin ∠UQR

= ½ x $10(\sqrt{3}-1)$ x 10 x sin 30

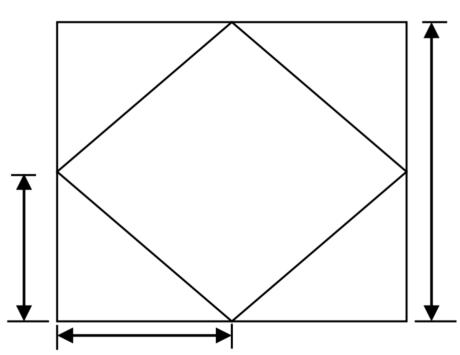
$= 50(\sqrt{3}-1) \times \frac{\sqrt{3}}{2}$

= 25 (3 - $\sqrt{3}$)

= (75 – 25$\sqrt{3}$) cm^2

$\therefore$ ar (ΔUQR) = = (75 – 25$\sqrt{3}$) cm^2

Q7.



64cm

32cm

32cm

∴ side of the 1st square = 64 cm

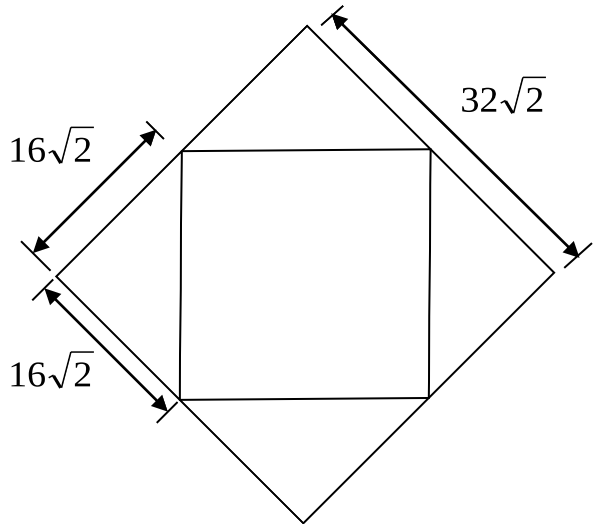
∴ side of the 2nd square $= \sqrt{32^2 + 32^2}$

$= \sqrt{2x32^2}$

$= \sqrt{2}\ x\ 32$

$= 32\ \sqrt{2}\ x\ \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{64}{\sqrt{2}}$



$\therefore$ Side of the 3rd square $= \sqrt{(16\sqrt{2})^2 + (16\sqrt{2})^2}$

$= \sqrt{2 + (16\sqrt{2})^2}$

$= (16\sqrt{2})\ x\ \sqrt{2}$

$= 32\ x\ \frac{2}{2}$

$= \frac{64}{2} = \frac{64}{(\sqrt{2})^2}$

$\therefore$ Side of the 1st square $= 64$ cm

$\therefore$ Side of the 2rd square $= \frac{64}{\sqrt{2}}$ cm

$\therefore$ Side of the 3rd square $= \frac{64}{(\sqrt{2})^2}$ cm

$\therefore$ In the similar fashion,

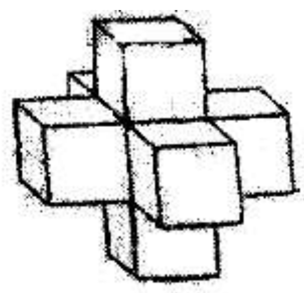
side of the 11^{th} square $\frac{64}{(\sqrt{2})^{10}}$

$= \frac{64}{2^5}$

$= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$

$=$ 2 cm

Q8.



Let, each side of a cube = a

$\because$ There are 7 cubes in the given solid,

$\therefore$ Total volume of the given solid = $7a^3$

$\therefore$ $7a^3 = 448$

$\Rightarrow a^3 = \frac{448}{7}$

$\Rightarrow a^3 = 64$

$\Rightarrow a = 3\sqrt{64}$

= 4 cm

∵ In the given solid,

We are able to see 5 surfaces of 6 cubes

∴ Total S.A. of the solid $= 5a^2 \times 6$

$= 30 \times 4^2$

$= 30 \times 16$

$= 480\ cm^2$

Q9. Let, us consider two cases, when Anil is a Rabbit and when Anil is a Fox,

CASE I :

When Anil is a Rabbit,

We have,

Anil is a Rabbit/Fox.

Chintoo is a Fox

Eashwar is a Fox

Bhauna is a Rabbit

Dolly is a Rabbit

In this case Anil may be a Rabbit or Fox.

∴ This is not possible

CASE II :

When Anil is a Fox

We have,

Anil is a Fox

Chintoo is a Rabbit

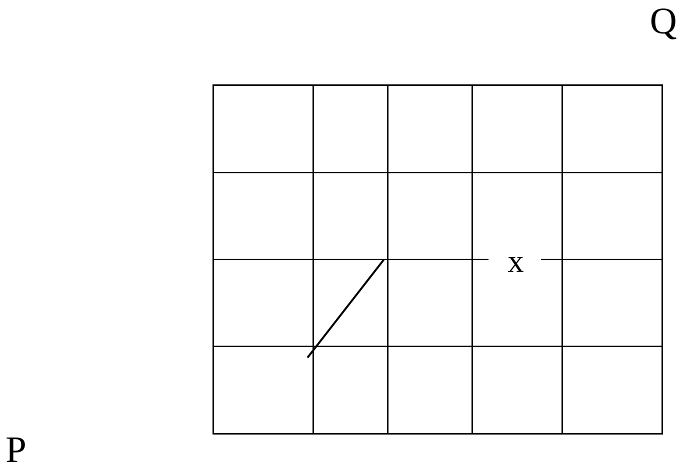
Eashwar is a Fox

Bhauna is a Fox

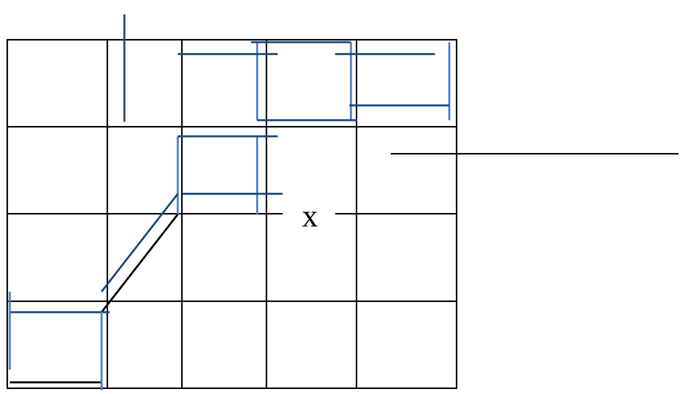
Dolly is a Fox

∴ There are 4 foxes.

Q10.



By using the principle of Pascal's triangle, we have,



$\therefore$ No. of shortest possible routes = 14