Indian National Physics Olympiad - 2008

INPhO-2008 Feb. 03, 2008
Duration: **4 Hours** Maximum Marks: **80**

Information:

1. This question paper consists of **5** pages.
2. There are **SEVEN (7)** questions and many of them are divided into subquestions.
3. All questions are compulsory.
4. Maximum marks for each sub-question and the whole question are indicated in brackets at the end of the question.
5. Use of log table and / or non-programmable electronic calculator is allowed.
6. Please carry out numerical computations carefully. Substantial marks will be deducted for numerical errors even if your method is correct.
7. In certain problems some of the later sub - questions can be successfully solved without solving the previous sub - questions. Be aware of this.
8. **Answer to each question should begin on a new page.**

No communication of any kind will be permitted among the candidates during the examination. Any query by the candidate is to be directed to the invigilator.

Table of Information

Speed of light in vacuum	c	$=$	3.00×10^{8} m-s^{-1}
Planck's constant	h	$=$	6.63×10^{-34} J-s
Universal constant of Gravitation	G	$=$	6.67×10^{-11} N-m^{2}-kg^{-2}
Magnitude of the electron charge	e	$=$	1.60×10^{-19} C
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} watt-meter^{-2} $-$ K^{4}
Permittivity constant	ϵ_0	$=$	8.85×10^{-12} F-m^{-1}
Permeability constant	μ_0	$=$	$4\pi \times 10^{-7}$ Henry-meter^{-1}

1. We define three quantities as follow:

$$A = m_e c^2,\ B = h/m_e c,\ C = e^2/2\epsilon_0 ch$$

Where m_e is electron mass and other symbols have their usual meanings. For the hydrogen atom, express the radius of the n^{th} Bohr orbit r_n, the energy level E_n, and the Rydberg constant R in terms of any two of $\{A,\ B,\ C\}$

[5]

2. Consider a ball which is projected horizontally with speed u from the edge of a cliff of height H as shown in the Fig. (1). There is air resistance proportional to the velocity in both x and y direction i.e. the motion in the x (y) direction has air resistance given by the $c\,v_x$ ($c\,v_y$) where c is the proportionality constant and $v_x(v_y)$ is the velocity in the x (y) directions. Take the downward direction to be negative. The acceleration due to gravity is g. Take the origin of the system to be at the bottom of the cliff as shown in Fig. (1).

   (a) Obtain expression for $x(t)$ and $y(t)$.

   (b) Obtain the expression for the equation of trajectory.

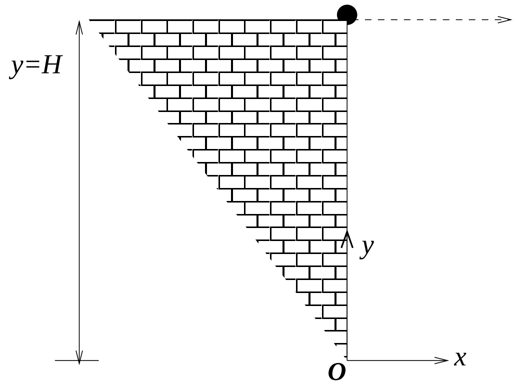

Figure 1:

(c) Make a qualitative, comparative sketch of the trajectories with and without air resistance.

(d) Given that height of cliff is 500 m and $c = 0.05$ sec^{-1}. Obtain the approximate time in which the ball reaches the ground. Take $g = 10$ m-sec^{-2}

[4 + 3 + 2 + 3 = 12]

3. Free Standing Tower

Consider a tower of constant density (ρ) and cross sectional area (A) (see Fig. (2)) at the earth's equator. The tower has a counter weight at one end. It is free standing. In other words its weight is balanced by the outward centrifugal weight so that it exerts no force on the ground beneath it and tension in the tower is zero at both ends. Consider the earth to be an isolated heavenly body and ignore gravitational effects due to the other heavenly bodies such as moon. Further assume that there is no bending of the tower.

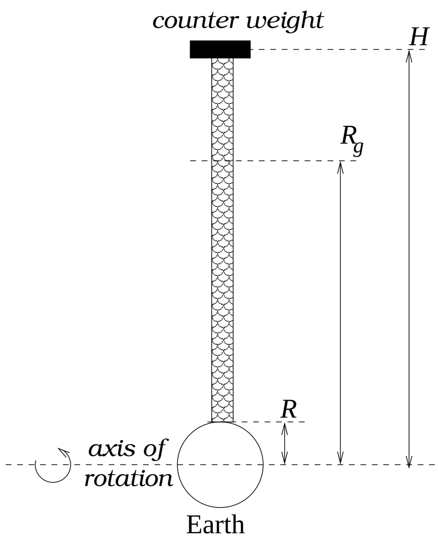

Figure 2:

(a) Draw the free body diagram of the small element of this tower at distance r from the center of the earth.

(b) Let $T(r)$ be the tensile stress (tension per unit area) in the tower. Use Newton's equations to write down the equation for dT/dr in terms of G, ρ, geostationary height R_g from the earth's center and earth's mass M.

(c) Taking the boundary condition ($T_R = T_H = 0$), obtain the height of tower H in terms of R and R_g. Note that R is the radius of earth. Calculate the value of H.

(d) The tensile stress in the tower changes as we move from $r = R$ to $r = H$. Sketch this tensile stress $T(r)$).

(e) Steel has density of $\rho = 7.9 \times 10^3$ kg-m^{-3}. The breaking tensile strength is 6.37 GPa. Calculate the maximum stress in the tower. State if a tower made of steel would be feasible.

Note: $M = 5.98 \times 10^{24}$ kg; $R = 6370$ km ; $R_g = 42\ 300$ km

[0.5 + 1.5 + 5 + 3 + 2 = 12]

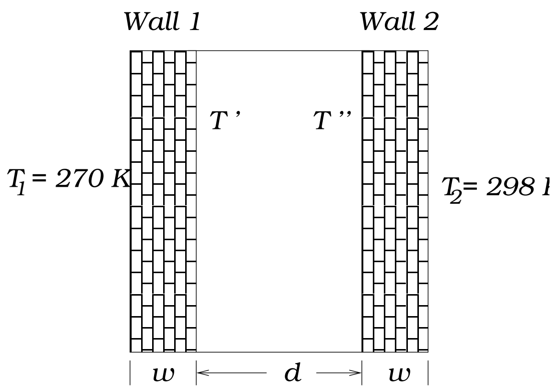

Figure 3:

4. Two identical walls, each of width w (=0.01 m), are separated by a distance d (= 0.1 m) as shown in Fig. (3). Temperatures of the external face of the walls are fixed (T_1 and T_2, $T_2 > T_1$). Coefficient of thermal conductivity of wall is $k_w = 0.72$ W-m^{-1}-K^{-1} . We define

$$T_0 = \frac{T_1 + T_2}{2}, \quad \Delta = T_2 - T_1 \quad \text{and} \quad \delta = T'' - T' \tag{1}$$

where T' and T'' are the temperatures of the internal face of the walls 1 and 2 respectively. Then δ will depend on the type of heat transfer process in central region (of width d) between the walls i.e. on the conduction, radiation or convection heat transfer. Assume that the heat transfer is a steady state process.

(a) Write down the expression for heat transfer flux q_w (watt-m^{-2}) inside the wall 1 in terms of k_w, T_1, T', and w. Similarly also write the expression for wall 2.

(b) Rewrite q_w in terms of Δ, δ, k_w, and w.

As mentioned above, in the central region between the walls, heat is transmitted by conduction, convection and radiation. Also due to the steady state process, the corresponding fluxes are equal to q_w. In what follows we will calculate the heat transfer fluxes between the walls due to these three processes each of these processes being considered separately.

Radiation process will take place without the presence of material medium in the central region between the walls. We assume that the central region between the walls is vacuum. Let ϵ be the emissivity of the walls and E_1 and E_2 be the total heat flux due to radiation from wall 1 to 2 and vice versa. Thus $E_1 = \epsilon\sigma T'^4 + (1-\epsilon)E_2$ where σ is Stefan constant. Similarly one may write the equation for E_2.

(c) The net heat transfer is $q_r = E_2 - E_1$. Write the expression for q_r in terms of ϵ, T'', and T'.

(d) Rewrite q_r in terms of $\{k_w,\ \Delta,\ T_0,\ \sigma,\ \epsilon$ and $w\}$.
[Hint: Eliminate δ using $\delta^2 << {T_0}^2$.]

(e) Calculate q_r if $\epsilon = 0.9$.

In the following two parts we are considering only convection betwen the walls.

(f) Now we assume that central region is filled with air of coefficient of thermal conductivity k_a. In this condition, convected heat transfer between walls will take place. Equation for flux due to this process is given by

$$q_{cv} = \frac{N_u\, k_a}{d}(T'' - T')$$

where N_u is called the Nusselt number and for the given system $N_u = 6.4$.
Due to the steady state nature of the process $q_w = q_{cv}$. Express q_{cv} in terms of $\{k_w,\ k_a,\ \Delta,\ w,\ d,$ and $N_u\}$.

(g) Calculate the value of q_{cv} if $k_a = 0.026$ W-m^{-1}-K^{-1}.

(h) Instead of air, the central region is now filled with sheathing material having coefficient of thermal conductivity k_s. Hence heat transfer will take place by conduction between walls. Express heat transfer flux q_{cd} in terms of $\{k_s,\ k_w,\ d,\ w,$ and $\Delta\}$. We assume that no radiation passes through sheathing material.

(i) Taking $k_s = 0.05$ W-m^{-1}-K^{-1}, calculate the value of q_{cd}.

(j) Considering all possible heat transfer process in the central region between the walls, which insulation (sheathing, air, or vaccum) is the most efficient?

[1 + 1 + 1 + 3 + 2 + 1.5 + 1.5 + 2.5 + 1.5 + 1 = 16]

5. Sunlight falls on the convex surface of the plano - convex lens of aperture 0.080 m. The radius of curvature of the convex surface of the lens is 0.100 m. The refractive indices of the material of the lens for extreme red and violet colours of sunlight are 1.600 and 1.700 respectively.

(a) Calculate the positions of the observed image of the Sun with violet and red center.

(b) Calculate the sizes of the observed image of the sun with violet and red center.

[3 + 7 = 10]

6. **Determination of The Speed of Light**:
The speed light maybe determined by an electrical circuit using low frequency ac fields only. Consider the arrangement shown in the Fig. (4). A sinusoidally varying voltage $V_0 \cos(2\pi f t)$ is applied to a parallel plate capacitor C_1 of radius a and separation s and also to the capacitor C_2. The charge flowing into and out of C_2 constitutes the current in the two rings of radii b and separation h. When the voltage is turned off the two sides (the capacitor C_1 on one side and the rings on the other) are exactly balanced. Ignore wire resistance, inductance and gravitational effects.

(a) Obtain an expression for the time-averaged force between the plates of C_1.

(b) Obtain an expression for the time-averaged force between the rings. The magnetic force between the two rings maybe approximated by those due to long straight wires since $b >> h$.

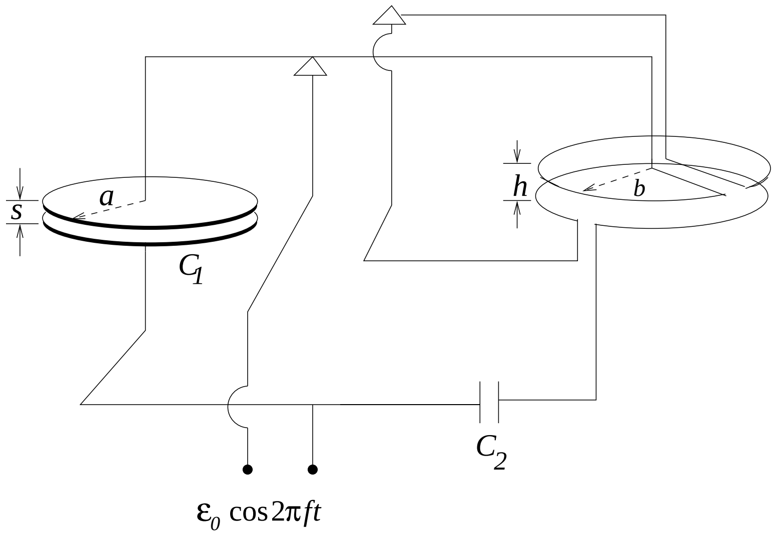

Figure 4:

(c) Assume that C_2 and the various distances are so adjusted that the time-averaged downward force on the upper plate of C_1 is exactly balanced by the time-averaged downward force on the upper ring. Under these conditions obtain an expression for the speed of light.

(d) Numerically estimate the speed of light given that: $a = 0.10$ m, $s = 0.005$ m, $b = 0.50$ m, $h = 0.02$ m, $f = 60.0$ Hz, $C_1 = 1.00$ nF (nano Farad) and $C_2 = 632$ μF (micro Farad).
[Hint: Not all the given quantities are required to obtain the estimate.]

[3 + 4 + 2 + 3 = 12]

7. An N turn metallic ring of radius a, resistance R, and inductance L is held fixed with its axis along a spatially uniform magnetic field $\vec{B}$ whose magnitude is given by $B_0 \sin(\omega t)$.

(a) Set up the emf equation for the current i in the ring.

(b) Assuming that in the steady state i oscillates with the same frequency ω as the magnetic field obtain the expression for i.

(c) Obtain the force per unit length. Further obtain its oscillatory part and the time-averaged compressional part.

(d) Calculate the time-averaged compressional force per unit length given that $B_0 = 1$ tesla, $N = 10$, $a = 10.0$ cm, $\omega = 1000.0$ radians/sec, $R = 10\ \Omega$, $L = 100$ mH.

(e) Answer the following two questions without providing rigorous justification:

i. For $\omega/2\pi = 60$ Hz, the ring emits a humming sound. What is the frequency of this sound.

ii. A capacitor is included in the circuit. How does this affect the force on the ring?

[2 + 3.5 + 4 + 1.5 + 2 = 13]

****** END OF THE QUESTION PAPER ******