Indian National Astronomy Olympiad – 2013

Question Paper   Roll Number: ______

INAO – 2013   Date: 2^{nd} February 2013
Duration: **Three Hours**   Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 4 pages (8 sides).
- There are total 10 questions. Maximum marks are indicated in front of each question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- No computational aides like calculators, log tables, slide rule etc. are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative schedule for OCSC in astronomy is from 2^{nd} to 19^{th} May 2013.

Useful Physical Constants

Mass of the Earth	M_E	$\approx$	6×10^{24} kg
Radius of the Earth	R_E	$\approx$	6.4×10^{6} m
Mass of the Sun	$M_\odot$	$\approx$	2×10^{30} kg
Radius of the Sun	$R_\odot$	$\approx$	7×10^{8} m
Radius of the Moon	R_m	$\approx$	1.7×10^{6} m
Distance to the Moon	d_m	$\approx$	3.84×10^{8} m
Mass of Jupiter	M_J	$\approx$	2×10^{27} kg
Astronomical Unit	1 A. U.	$\approx$	1.5×10^{11} m
Light year	1 Ly	$\approx$	9.461×10^{15} m
	1 Ly	$\approx$	63240 A.U.
Solar Luminosity	$L_\odot$	$\approx$	3.826×10^{26} W
Gravitational Constant	G	$\approx$	6.67×10^{-11} m^3/(Kg s^2)
Gravitational acceleration	g	$\approx$	9.8 m/s^2
Wien's constant	K	$\approx$	2.898×10^{-3} m.K

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

1. (10 marks) Astrophysicist Freeman Dyson proposed around 50 years back that an advanced civilisation would make optimal use of the energy of the parent star by constructing a full shell around the parent star with radius equal to the orbital radius of their planet, trapping all the radiation inside. The civilisation can live on the surface of this shell. We would like to construct a "Dyson sphere" with radius equal to the orbital radius of the Earth. Let us assume that we have access to all the material within the solar system (except the Sun itself, which we would like to retain as the energy source) and we have necessary technology to solidify the material available to us. What will be rough thickness of this shell?

Solution: Total mass available to us for constructing the Dyson Sphere would primarily consist of mass of all planets and satellites. As an approximation, we will say Jupiter and Saturn are of similar size. Saturn is known to have very low density so its mass will not exceed half Jupiter mass. Further, radius of Uranus and Neptune is less than half of Jupiter radius. Thus, they will be at least 10 times lighter than Jupiter. Similarly, Earth and Venus are of similar size and Mars is about half of that. There are a handful of objects with radii about a quarter of that of Earth or even smaller (Mercury, Moon, Ganymede, Callisto, Titan etc.). All these are rocky bodies and we can take typical density to be $\rho_s = 5$ gm/cc. Combined mass of all other combined bodies will not be greater than mass of Mars.

From real planetary data, total mass estimate of about $1.5 M_J$ is closer to reality.

As we want shell to be solid like rock, we will assume similar density for the shell too. So if the shell has thickness l,

$$
\begin{aligned}
4\pi D^2 l \rho_s &\approx \left[2M_J + 2\frac{M_J}{10} + 2M_\oplus + 2\frac{M_\oplus}{10} + 6\frac{M_\oplus}{100}\right] \\
\therefore l &\approx \frac{2M_J + 0.2M_J + 2M_\oplus + 0.2M_\oplus + 0.06M_\oplus}{4\pi D^2 \rho_s} \\
&\approx \frac{2.2(M_J + M_\oplus)}{4\pi (1.5\times10^{11})^2 \times 5 \times 10^3} \\
&\approx \frac{2.2(2\times10^{27} + 6\times10^{24})}{4\pi \times 2.25\times10^{22} \times 5\times10^3} \\
&\approx \frac{2006}{200\pi} \approx \frac{10}{\pi} \\
&\approx 3 \text{ mt.}
\end{aligned}
$$

Thus, the thickness will be approximately of the order of a few meters.

As is evident from the numbers, mass estimates are just rough indicators of total mass. and density can be lowered to about 2.5 gm/cc. Thus, the thickness can go up to 5 metres.

Marching Scheme:
Reasonable estimate of mass ($\approx 1.5 - 3.0 M_J$) and density (1 - 10 gm/cc) with

reasons: 5 marks
Rest of the Calculation: 5 marks

2. (10 marks) Following table contains names of a few constellations and names of some stars. Fill in the blanks. For missing star names, you can name any star from that constellation.

STAR	CONSTELLATION
Polaris	—
Rigil Kentaurus	—
—	Orion
—	Cygnus
—	Taurus
Sirius	—
Vega	—
Regulus	—
—	Virgo
—	Scorpio

Solution:

STAR	CONSTELLATION
Polaris	**Ursa Minor**
Rigil Kentaurus	**Centaurus**
Betelgeuse	Orion
Deneb	Cygnus
Aldebaran	Taurus
Sirius	**Canis Major**
Vega	**Lyra**
Regulus	**Leo**
Spica	Virgo
Antares	Scorpius

3. (10 marks) An Indian festival called 'Kojagiri' is always celebrated on a Full Moon night in Autumn. As a part of festivities, many people keep a bowl of milk on the ground or a terrace under the Moon light for a few minutes, before drinking it. On this year's 'Kojagiri' night, Sheetal had placed roughly 100 ml milk in a dish of size 25cm diameter for about 10 minutes exactly when the moon was overhead. Assume that density of milk is similar to the water, its specific heat capacity is 0.9 times that of water and heat loss from the milk to surroundings is negligible. Assume that full Moon reflects about 14% of the sunlight incident on its surface. What will be the change of temperature caused by the light received from the moon?

Solution: Moon receives light energy from the Sun which is partly reflected back towards the Earth.

$$\begin{aligned}
\text{Solar Energy per unit area incident on Moon} &= \frac{L_\odot}{4\pi D^2} \quad \text{(1 mark)} \\
\text{Total Solar Energy incident on Moon} &= \frac{L_\odot}{4\pi D^2} \times \pi R_m^2 \quad \text{(1 mark)} \\
\text{Total Solar Energy reflected by Moon} &= \frac{L_\odot}{4\pi D^2} \times \pi R_m^2 \times \alpha \quad \text{(1 mark)} \\
\text{Reflected Energy received at Earth per unit area} &= \frac{\pi L_\odot R_m^2 \alpha}{4\pi D^2 \times 2\pi d_m^2} = E_r \quad \text{(2 marks)}
\end{aligned}$$

Let us assume that this entire energy is used to heat the milk without any losses.

$$\begin{aligned}
tE_r A &= ms\Delta T \quad \text{(1 mark)} \\
\Delta T &= \frac{t\pi r^2}{V\rho s} \times \frac{\pi L_\odot R_m^2 \alpha}{4\pi D^2 \times 2\pi d_m^2} \quad \text{(1 mark)} \\
&= \frac{600 \times 0.125^2 \times 3.826 \times 10^{26} \times (1.7 \times 10^6)^2 \times 0.14}{10^{-4} \times 10^3 \times 0.9 \times 4.18 \times 10^3 \times 8 \times (1.5 \times 10^{11})^2 \times (3.84 \times 10^8)^2} \quad \text{(1 mark)} \\
\Delta T &\approx \frac{1}{7000} \approx 1.4 \times 10^{-4} \quad \text{(2 marks)}
\end{aligned}$$

Thus, temperature will rise by only about 10^{-4} Celsius.

4. (7 marks) Let A and B be two objects in the solar system orbiting each other and A is much heavier than B. Theoretically, what is the maximum possible revolution period of B?

Solution: To have longest possible period, the orbital radius should also be longest. If A was any planet, orbital radius of B cannot be very high as far from the planet B will come under influence of the Sun. So for longest possible radii, we take A as the Sun and try to place B as far as possible. (1.5 marks)

The other limiting factor will be the influence of other stars. As B is still part of the solar system, Sun exerts more influence on it than any other star. (1.5 marks)

Star closest to us is the Alpha-Centauri triplet. It has two stars similar to the Sun and Proxima Centauri, which is much smaller than the Sun. Thus, we can approximate the system as about $2M_\odot$ system about 4.3 light years away. Hence, at about $\frac{1}{3}^{\text{rd}}$ of the distance, Alpha-Centauri system will start dominating on B. This means that maximum possible radius is about $\frac{4.3}{3} = 1.4$ light years. Thus, longest period will be, (1.5 marks)

We can utilise the fact that orbital radius of the Earth is 1 A.U. and orbital period is 1 year.

$$
\begin{aligned}
T^2 &= \frac{4\pi^2}{GM} r^3 \quad \text{(0.5 marks)} \\
\frac{T_1^2}{T_2^2} &= \frac{R_1^3}{R_2^3} \\
T_1 &= T_2 \sqrt{\frac{R_1^3}{R_2^3}} \\
&= 1 \times \sqrt{\frac{(1.4 \times 63240)^3}{1^3}} \\
&\approx \sqrt{(90000)^3} \\
&= 300^3 \\
&= 2.7 \times 10^7 \text{ years} \quad \text{(2 marks)}
\end{aligned}
$$

Thus, approximate period would be 27 million years.

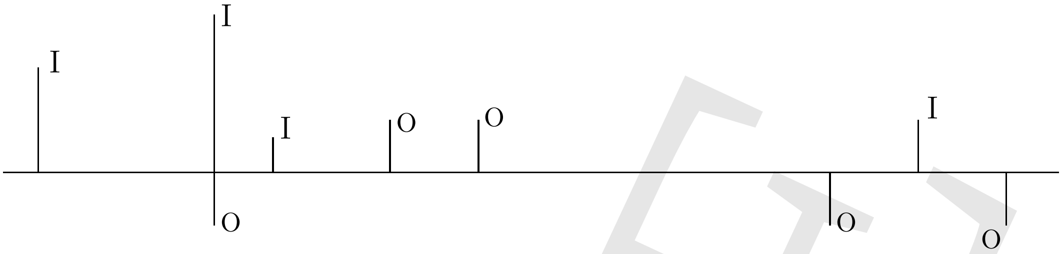
It is acceptable if Alpha-Centauri system is assumed to have mass of just a $M_\odot$.

5. (8 marks) Prove that $2013 \times 2012^4 + 1$ is a composite number.

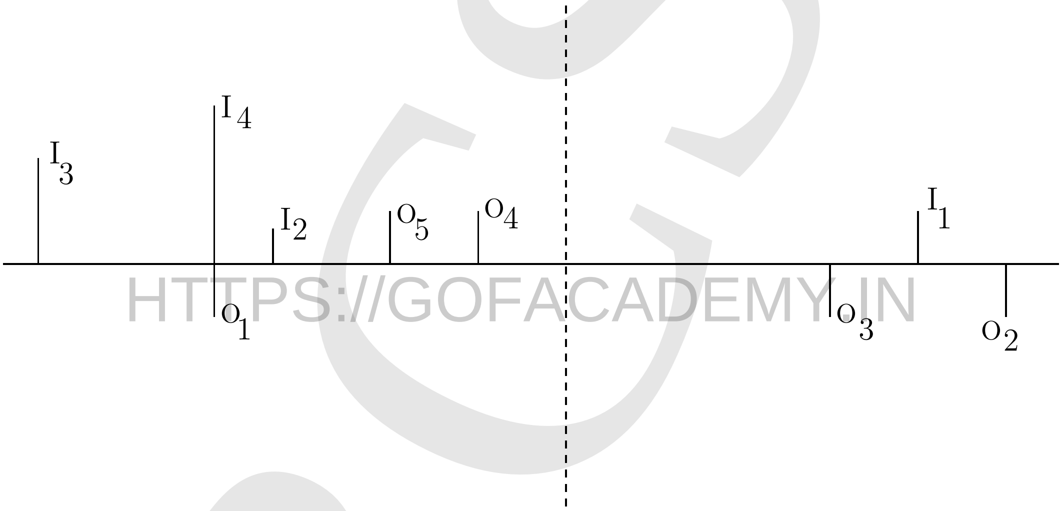
Solution:

$$
\begin{aligned}
X &= 2013 \times 2012^4 + 1 \\
&= (2012 + 1) \times 2012^4 + 1 \\
&= (a + 1) \times a^4 + 1 \text{ where } a = 2012 \\
&= a^5 + a^4 + 1 \quad \text{(2 marks)} \\
&= a^5 + a^4 + a^3 - a^3 + 1 \\
&= a^3(a^2 + a + 1) - (a^3 - 1) \\
&= a^3(a^2 + a + 1) - (a - 1)(a^2 + a + 1) \\
&= (a^2 + a + 1)(a^3 - a + 1) \\
&= (2012^2 + 2013)(2012^3 - 2011) \quad \text{(6 marks)}
\end{aligned}
$$

6. (10 marks) Ayush was doing an experiment with a lens in his Physics lab. He kept a reference object at several positions and noted corresponding image sizes and positions. The same are shown in the diagram below. Unfortunately, he forgot to label the object and image positions and to mention details about the lens, because he thought them to be too trivial. Help his science teacher to make sense of his drawing by finding type of lens (convex / concave), its position and its focal length. In the diagram, O stands for Object and I for image. All lengths and heights are to scale.



Solution: The type of lens is convex lens. In the picture, Object-Image pairs have been given same subscript. O_5 is placed at exact focus so it has no corresponding image. Focal length of lens is about 2.25 cm (From the figure, $4f = 9.1$ cm).



Marking scheme: Lens type - 1 mark, Reasoning - 2 marks, position - 3 marks, focal length - 3 marks, cross checking - 1 mark

7. (5 marks) In a competition for the most useless inventions, Sharad presented a novel technique which makes it possible to fold an ordinary paper 50 times on itself, without any gaps between layers of folds. Calculate thickness of the folded paper after it has been folded 50 times. Note that the paper is folded further and further without opening the previous folds i.e. it is folded in half and then the folded paper is folded on the middle to reduce the area to a quarter and so on.

Solution: A 100 page notebook is roughly 1 cm thick. (1 mark)
Thus, thickness of a single paper is 10^{-2} cm or 10^{-4} mt ($2 \times 10^{-4} - 2 \times 10^{-5}$ mt accepted). (1 mark)

If you fold the paper 50 times, its thickness will become,

$$
\begin{aligned}
l &= 2^{50} \times 10^{-4} \quad \text{(2 marks)} && (1)\\
&= \left(2^{10}\right)^5 \times 10^{-4} = (1024)^5 \times 10^{-4} && (2)\\
&\approx 10^{15} \times 10^{-4} && (3)\\
&\approx 10^{11}\ \text{mt.} \quad \text{(1 mark)} && (4)
\end{aligned}
$$

Thus, the thickness will be of the order of the Sun-Earth Distance.

8. (10 marks) Alankar was jogging downhill and he noted distance traveled by him at various intervals. Plot appropriate graph to find his acceleration and initial velocity. In the table, time (t) is in minutes and distance (d) is in meters. Both the quantities are measured from the start of the journey.

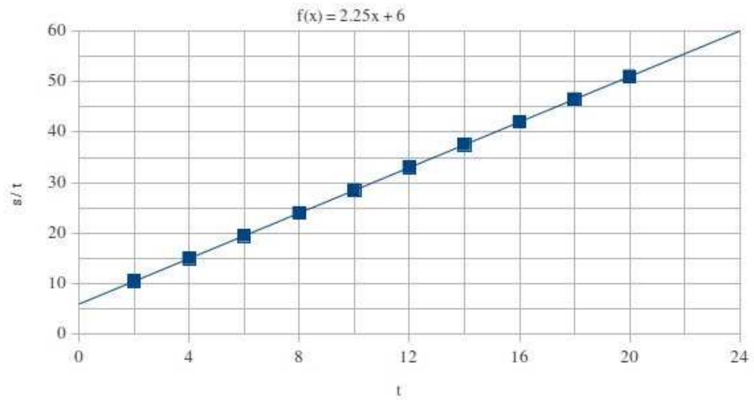
t	d	t	d	t	d	t	d	t	d
2	21	6	117	10	285	14	525	18	837
4	60	8	192	12	396	16	672	20	1020

Solution: $d = ut + \frac{1}{2}at^2$

Thus, if one plots a graph of distance versus time, it will be a parabolic curve. To linearise the graph, we change the equation as $\frac{d}{t} = u + \frac{a}{2}t$

Now if we plot $\frac{d}{t}$ versus t, it will give a linear graph with $\frac{a}{2}$ as slope and u as y-intercept.

t	d/t	t	d/t	t	d/t	t	d/t	t	d/t
2	10.5	6	19.5	10	28.5	14	37.5	18	46.5
4	15	8	24	12	33	16	42	20	51

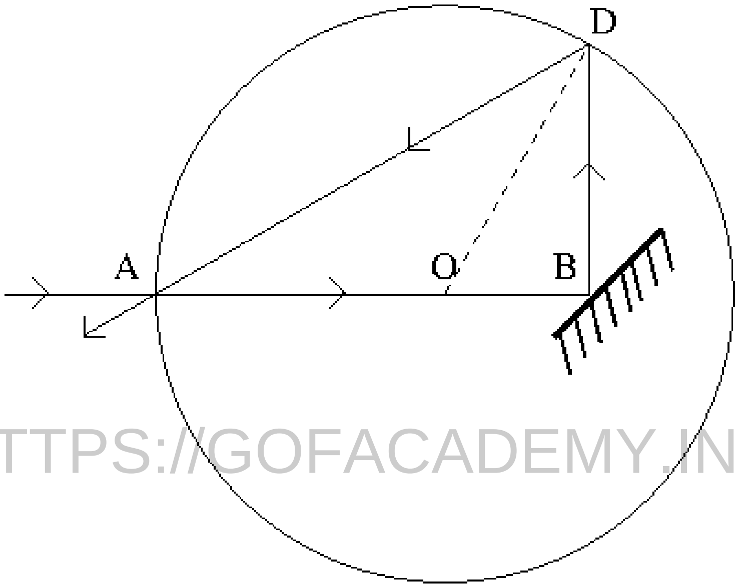


Thus, $u = 6$ m/min and $a = 4.5$ m/min^2.

Marking scheme: Linearisation of equation: 2 marks, graph plotting skills: 4 marks, slope: 2 marks, final answers: 2 marks. Solutions with non-linear graphs: maximum 5 marks.

9. (10 marks) Sandesh fabricated a magic sphere of radius R, which is hollow on the inside and has perfectly reflecting inner surface. This sphere had a small hole in it. Sandesh sent a ray of light radially through this hole. It hit a plane mirror, kept at an angle of 45° with the incident ray, at some point beyond the centre of the sphere, but before reaching the opposite end. After undergoing one more reflection at the inner surface of the sphere, the ray came out from the hole. Find the distance from the centre of the sphere to the point where it struck the plane mirror.

Solution: In the figure below, D is the point of second reflection. $\overline{OD}$ will be normal to the surface at D.



$$\begin{aligned}
\measuredangle OBD &= 90^\circ \quad \text{(1 mark)} \\
\measuredangle BDO &= \measuredangle ADO = \measuredangle DAO = x \quad \because AO = OD \quad \text{(6 marks)} \\
\therefore \measuredangle AOD &= 180 - (\measuredangle ADO + \measuredangle DAO) \\
&= 180 - 2x \\
\text{but } \measuredangle AOD &= \measuredangle OBD + \measuredangle BDO \\
180 - 2x &= 90 + x \\
\therefore x &= 30 \quad \text{(2 marks)} \\
l(OB) &= l(OD)\sin 30 = \frac{R}{2} \quad \text{(1 mark)}
\end{aligned}$$

10. (20 marks) Answer following questions in 3 to 4 lines each.
   (a) The core of earth is like a perfectly conducting fluid sphere. If this core suddenly shrinks to half of its present radius, then what will happen to its magnetic field? Why?

(b) A satellite is revolving around the Earth in a polar orbit with 90 minute period. On 23rd September, Prasad saw it exactly overhead at the time of sunset. Within next twenty four hours how many sunrises will be seen by the satellite? Why?

(c) Anand has taken a high zoom photograph of a sun-spot. The photograph does not include any part of the disk outside sun-spot. What will be the predominant colour seen in the photograph? Why?

(d) What will happen to the Moon if the Earth vanishes suddenly? Why?

(e) On a full Moon day, with respect to an observer on the Earth, does the Moon move faster (angle covered per hour) during the day time or the during night time? Why?

Solution: (a) Magnetic flux must be conserved. Thus, $B \propto \dfrac{1}{R^2}$. As the radius shrinks to half, the magnetic field will increase four-fold.

(b) As the satellite makes angle of 90° with the Sun, its orbit is exactly along the line separating day and night regions on the Earth's surface. Hence, its one side will continuously face the Sun and hence, there will be NO sunrises seen by the satellite.

(c) The sunspots have temperature of around 4500K. By Wien's law, this corresponds to roughly red colour. Thus, the sunspots will appear red.

(d) Moon will retain all its velocity at the instance of disappearance of the Earth. Thus, it will have significant non-radial velocity around the Sun and it will continue to orbit the Sun in near identical path.

(e) We note that Moon revolves in the same direction as the rotation of the Earth i.e. from West to East. On the full Moon day, Moon is on the night side of the Earth. Thus, with respect to an observer on the Earth, the tangential velocities of the Moon and the Earth during the night will be in the same direction and hence relative velocity is slower. During the day tangential velocities are in opposite directions hence the relative velocity is faster. Thus, Moon will be moving faster during the day.

Space for Rough Work

Notes for Junior Group

- Question 4: Take body A to be the Sun. Kepler's third law states that the square of orbital period will be proportional to the cube of orbital radius.
- Question 5: $a^3 - 1 = (a-1)(a^2 + a + 1)$.
- Question 8: Modify appropriate equation of motion such that all points will lie along a straight line.
- Question 10 (a): If the conducting sphere suddenly changes the size, the field lines will stay trapped on the surface.
- Question 10 (c): At any given temperature, the peak of thermal radiation will be at wavelength $\lambda_{max} = \frac{\text{Wien's constant}}{T}$.

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